Total Number of Questions: 20
Max. Marks: 100

Time: 2.00 Hours

1. Discuss convergence in probability, convergence in distribution and convergence in Mean and their relationships. **(6 marks)**

2. Explain and compare the fundamental concepts and properties of probability generating functions, moment generating functions and characteristics functions in the context of probability theory and statistical analysis. **(6 Marks)**

3. Define sufficiency and minimal sufficiency, and explain their significance in the context of estimation. Provide examples to illustrate how sufficiency is determined and how it influences the selection of estimators. **(6 Marks)**

4. Define Fisher information, discuss its role in quantifying the amount of information in a sample, and explain how it relates to the Cramer- Rao inequality. **(6 Marks)**

5. Describe Mann- Whitney U test. **(4 marks)**

6. Explain briefly the Latin Square- Model in design of experiment. **(4 marks)**

7. Find the best critical region to test the null $H_0 : \theta = 0$ against the alternate $H_1 : \theta = 1$ for a population with pdf $f(x, \theta) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}}$, $-\infty < x < \infty$ **(6 Marks)**

8. Derive the test statistic for testing the significance of the difference of proportions of a certain attribute of two populations, when the sample size is large. **(6 Marks)**

9. In a simple random sample of 200 from a population of 2400 colleges, 120 colleges were in favour of a proposal. Estimate the total number of colleges in the population that favoured the proposal and find the variance of this estimate. **(4 Marks)**

10. If X_1 and X_2 are two independent random variables with X_i follows Binomial $(n_i, \frac{1}{2})$ for $i = 1, 2$. Find the distribution of $X_2 - X_1 + n_1$ **(4 Marks)**

11. Consider a systematic sampling of selecting n units from a population of N units, where $N = nk$. Let S^2_{wsy} be the variance among units within the same systematic sample and S^2 be the variance as a whole. Obtain the condition for which mean of the systematic Sample $\bar{y}_{sy}$ is more efficient than the mean of the simple random sample $\bar{y}$ of same size. **(6 Marks)**

12. Let X and Y are two independent $N(0,1)$ random variables. Obtain the distribution of
$W = \dfrac{X}{|Y|}$. (6 Marks)

13. Let X Be distributed as $N_p(\mu, \varepsilon)$. Write down the characteristic function of X and hence find the distribution of CX, where C is no-singular. Explain the procedure of estimating the parameter μ. (4 Marks)

14. Distinguish between Hotelling T^2 in two sample problem and Mahalonobis D^2 Statistics and obtain the interrelationship between them. What are they primarily used for? (4 Marks)

15. Define recurrent state of a Markov chain. Prove or disprove: A state 'i' is recurrent if and only if $\displaystyle\sum_{n=1}^{\infty} P_{ii}^n$ is divergent. (4 Marks)

16. Carry out the distinction between Poisson, pure birth and birth- death processes. (6 Marks)

17. The least square estimator $\hat{\beta}_0$ is an unbiased estimator of the model parameter β_0 in fitted regression model. Prove (4 Marks)

18. $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Find A^3 by using Cayley- Hamilton theorem. (4 Marks)

19. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$ then prove $fg \in \mathcal{R}(\alpha)$. (4 Marks)

20. In a logistic Regression model the odds ratio can be interpreted as the estimated increase in the probability of success associated with a one unit change in the value of the predictor variable. Prove (6 Marks)