1. The matrix equation $X^2 = I_2$, where I_2, the identity matrix of order 2 and X is a square matrix of order 2 over the field of real numbers has :
   (A) No solutions
   (B) One solution
   (C) Two solutions
   (D) Infinitely many solutions

2. Let $G = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b, \in \mathbf{R} \right\}$ and let + and . denote the matrix addition and multiplication respectively. Then ________.
   (A) (G, +) is a group but (G, .) is not a group.
   (B) (G, .) is a group but (G, +) is not a group.
   (C) (G, .) is isomorphic with (C, .) where C is the group of complex numbers.
   (D) None of the above

3. Let D be the group of symmetries of the square. Then ________.
   (A) D has 4 subgroups of order 2
   (B) D has 3 subgroups of order 4
   (C) Order of D is 16
   (D) Order of D is 12

4. Which of the following sets is a subspace of the vector space $\mathbf{R}^2$?
   (A) $\{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 1\}$
   (B) $\{(x, y) \in \mathbf{R}^2 : xy = 1\}$
   (C) $\{(x, y) \in \mathbf{R}^2 : y = x\}$
   (D) $\{(x, y) \in \mathbf{R}^2 : y = x^2\}$

5. G is a group of order 31. Then ________.
   (A) G is cyclic
   (B) G is non-cyclic
   (C) G is non-abelian
   (D) None of the above

6. Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined by $T(x, y, z) = (x, y, 0)$. Then ________.
   (A) (0, 0, 4) is in the range space.
   (B) $\left(2, \frac{1}{\sqrt{2}}, 0\right)$ is in the zero space.
   (C) (1, 1, 1) is in the range space.
   (D) (0, 0, 1) is in the zero space.

7. The geometrical effect of the linear transformation associated with the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ is :
   (A) a rotation by an angle $\frac{\pi}{2}$
   (B) stretching along Y-axis and a reflection with respect to Y-axis
   (C) a stretching along X-axis
   (D) a reflection with respect to X-axis

8. If T : V → V is an orthogonal transformation on an inner product space V and if
(I) : T is an isometry
(II) : T takes orthonormal basis to orthonormal basis then ________.
(A) (I) is true but (II) is false
(B) (I) is false but (II) is true
(C) Both (I) and (II) are true
(D) Both (I) and (II) are false

9. Let * be the cross product in the Euclidean space R^3. Then ________.
(A) * is associative
(B) * is commutative
(C) * is not associative
(D) None of the above

10. Choose the linear mapping from the following :
(A) $F : R^2 \to R^2$ defined by $F(x, y) = (x^2, y^2)$
(B) $F : R^3 \to R^2$ defined by $F(x, y, z) = (x + 2y - 3z, 4x - 5y + 6z)$
(C) $F : R^3 \to R^2$ defined by $F(x, y, z) = (|x|, y + z)$
(D) $F : R^2 \to R^2$ defined by $F(x, y) = (xy, y)$

11. Domain of the function $y = \sqrt{1 - x^2}$ is :
(A) R
(B) [0, 1]
(C) [−1, 1]
(D) (0, 1)

12. $\oint xy\,dy - y^2\,dx$ is :
(A) 0
(B) $\frac{3}{2}$
(C) 1
(D) π

13. Laplace equation $U_{xx} + U_{yy} = 0$ is :
(A) Elliptic
(B) Parabolic
(C) Hyperbolic
(D) None of these

14. Number of different permutations of 'n' items taken 'k' at a time without repetition is :
(A) n!
(B) $\frac{n!}{k!}$
(C) $\frac{n!}{(n-k)!}$
(D) 2^n

15. Every polynomial of degree ≥ 1 has ________ zero.
(A) exactly one
(B) infinitely many
(C) atleast one
(D) none of these

16. Upper triangular matrices are :
(A) Matrices having non-zero entries above the main diagonal.
(B) Square matrices having non-zero entries above the main diagonal.
(C) Matrices having non-zero entries only on and above the main diagonal.
(D) Square matrices having non-zero entries only on and above the main diagonal.

17. The Wronskian of functions $\sin x$ and $\cos x$ is :
(A) −1
(B) 0
(C) 1
(D) π

18. An operator T on a Hilbert space H is normal, then it's adjoint T^* is ________.
(A) a linear transformation
(B) a polynomial in T
(C) orthogonal
(D) none of these

19. Unit tangent vector of the helix $s(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$ is :

(A) sin t

(B) $\frac{\sin t}{\sqrt{2}}\,\hat{i} + \frac{\cos t}{\sqrt{2}}\,\hat{j} + \frac{t\hat{k}}{\sqrt{2}}$

(C) $\frac{\sin t}{\sqrt{2}}\,\hat{i} + \frac{\cos t}{\sqrt{2}}\,\hat{j} + \frac{1}{\sqrt{2}}\,\hat{k}$

(D) none of these

20. $\int_0^{\pi/2} \log \tan x \, dx$ is :

(A) 1 (B) e (C) 0 (D) π

21. If $x > 0$ and if y is an arbitrary real number, then there is a positive integer n such that $nx > y$. This property of real number is called :

(A) Supremum property
(B) Archimedean property
(C) Approximation property
(D) Comparison property

22. Let A be the set of all sequences whose elements are the digits 0 and 1. Then the set A is :

(A) Finite
(B) Countable
(C) Uncountable
(D) Having 2^n elements

23. The $\lim_{n \to \infty} \left((2n)^{1/n}\right)$ is :

(A) 0 (B) 2 (C) 2 e (D) 1

24. $\lim_{x \to 0} x \cos\left(\frac{1}{x}\right)$ is :

(A) 0 (B) Does not exist (C) 1 (D) ∞

25. The series $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots\cdots\cdots$

(A) Tends to ∞ (B) Converges (C) Diverges (D) Oscillates

26. Which of the following functions is uniformly continuous ?

(A) $f(x) = \frac{1}{x}$ for $x \in [1, \infty)$

(B) $f(x) = \frac{1}{x},\ x \in (0, 1)$

(C) $f(x) = x^2,\ x \in \mathbf{R}$

(D) $f(x) = \sin\left(\frac{1}{x}\right),\ x \in \mathbf{R}$ such that $x > 0$

27. Let f be a function defined on [0, 1] by
$f(x) = 0$ when x is rational
$= 1$ when x is irrational
The Lebesgue integral of f over [0, 1] is :
(A) 0 (B) Does not exist (C) ∞ (D) 1

28. Suppose $f \geq 0$ and f is continuous on [a, b] and $\int_a^b f(x)\,dx = 0$. Which of the following is true ?
(A) $f(x) = 0$ for all $x \in [a, b]$
(B) $f(x) = 1$ for all $x \in [a, b]$
(C) f is monotonic increasing on [a, b]
(D) Image of f is $[0, \infty)$

29. Which of the following is true ?
(A) Every Lebesgue integrable function is Riemann integrable
(B) Every Riemann integrable function is Lebesgue integrable
(C) Limits of Riemann integrable functions are always Riemann integrable
(D) Limits of measurable functions need not be measurable

30. The measure of the cantor set is :
(A) ∞ (B) 1 (C) 0 (D) not measurable

31. If $\omega \neq 1$ is an n^{th} root of unity, then $1 + 2\omega + 3\omega^2 + ... + n\,\omega^{n-1} =$
(A) 0 (B) $\frac{1}{(1-\omega)^2}$ (C) $\frac{-n}{\omega - 1}$ (D) $\frac{n}{\omega - 1}$

32. Which one of the following functions is not the real part of an analytic function ?
(A) $u(x, y) = x^2 - y^2$ (B) $u(x, y) = x^2 + y^2$
(C) $u(x, y) = x^3 - 3xy^2 - y$ (D) $u(x, y) = e^x \cos y$

33. Which one of the following is not true about $f(z) = \sin z$?
(A) it is a bounded function (B) it is an entire function
(C) it is a periodic function (D) its zeroes are isolated

34. Choose the correct statement :
(A) $f(z) = e^{1/z}$ has an essential singularity at $z = \infty$
(B) $f(z) = \frac{e^z - 1}{z}$ has a simple pole at $z = 0$
(C) $f(z) = z^2 + 1$ has a pole of order 2 at $z = \infty$
(D) $f(z) = \frac{1}{\sin\frac{1}{z}}$ has an isolated singularity at $z = 0$

35. Let C be the circle $|z|=1$. Then,

$$\int_C e^{4/z}\, dz =$$

(A) $4\pi i$ (B) $-4\pi i$ (C) $-8\pi i$ (D) $8\pi i$

36. Consider the metric space (M, d), where $M=[0, 1]$ and d is the usual metric. Then the open ball $B\left(0, \frac{1}{2}\right)=$

(A) $\left[0, \frac{1}{2}\right)$ (B) $\left(0, \frac{1}{2}\right)$ (C) $\left(0, \frac{1}{2}\right]$ (D) $\left[0, \frac{1}{2}\right]$

37. The topology on R induced by the Euclidean metric is called :
(A) discrete topology
(B) indiscrete topology
(C) usual topology
(D) sorgenfrey topology

38. Choose the correct statement :
(A) Every connected space is path connected.
(B) Local connectedness is a hereditary property.
(C) Every path connected space is connected.
(D) Local connectedness is preserved under continuous functions.

39. Which one of the following is not a divisible property ?
(A) Being a discrete space
(B) Being a locally connected space
(C) Being a finite space
(D) Being a regular space

40. Choose the incorrect statement :
(A) Every second countable space is Lindeloff.
(B) Every subspace of a normal space is normal.
(C) Every metric space is normal.
(D) Every compact subset in a Hansdorff space is closed.

41. $\beta(m, n)$ is equal to :

(A) $\frac{\Gamma m+\Gamma n}{\Gamma(m+n)}$ (B) $\frac{\Gamma m-\Gamma n}{\Gamma(m-n)}$ (C) $\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$ (D) $\frac{\Gamma(m+n)}{\Gamma m\Gamma n}$

42. According to Rodrigue's Formula :

(A) $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2+1\right)^n$ (B) $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$

(C) $P_n(x)=\frac{n}{2^n}\frac{d^n}{dx^n}\left(x^2+1\right)^{-n}$ (D) $P_n(x)=\frac{n-1}{2^n}\frac{d^n}{dx^n}\left(x^2-1\right)^{-n}$

43. $L\left(\frac{1-\cos t}{t}\right)$ is :

(A) $\frac{1}{2}\log\left\{\frac{(s^2+1)}{s^2}\right\}$

(B) $-\frac{1}{2}\log\left\{\frac{(s^2+1)}{s^2}\right\}$

(C) $\frac{1}{2}\log\left\{\frac{s}{s^2+1}\right\}$

(D) $-\frac{1}{2}\log\left\{\frac{s}{s^2-1}\right\}$

44. The solution of $(12x+5y-9)dx+(5x+2y-4)dy=0$ is :

(A) $6x^2-5xy-y^2+9x-4y=c$

(B) $3x^2-4xy+9x-3y=c$

(C) $6x^2+5xy-y^2-9x-4y=c$

(D) $6x^2+5xy+y^2-9x-4y=c$

45. In the Fourier series expansion for the function x^2 in the interval $[-\pi, \pi]$ the value of a_n is :

(A) $\frac{2(-1)^n}{n}$

(B) $\frac{4(-1)^n}{n^2}$

(C) $\frac{4}{n^2}$

(D) $\frac{2(-1)^n}{n^2}$

46. Solution of the Clairaut's equation $p=\log(px-y)$ where $p=\frac{dy}{dx}$ is :

(A) $y=cx$

(B) $c=\log(cx-y)$

(C) $(y-cx)(c-1)=1$

(D) $(y-1)c=c-1$

47. With usual notations $F(a, b; c; y)$ is given by :

(A) $1-\sum_{n=1}^{\infty}\frac{(a)_n(b)_n y^n}{(c)_n n!}$

(B) $\sum_{n=1}^{\infty}\frac{(a)_n(b)_n y^n}{(c)_n n!}-1$

(C) $1-\sum_{n=0}^{\infty}\frac{(a)_n(c)_n y^n}{(b)_n n!}$

(D) $1+\sum_{n=1}^{\infty}\frac{(a)_n(b)_n y^n}{(c)_n n!}$

48. The equation $(1-x^2)y''-2xy'+n(n+1)y=0$ is called ________.

(A) Bessel equation

(B) Legendre equation

(C) Chebyshev's equation

(D) Laguerre equation

49. In the 4th order Runge-Kutta method the global error will be :

(A) $O(h^2)$

(B) $O(h^3)$

(C) $O(h^4)$

(D) $O(h^6)$

50. Using Picard's method the approximate solution to the initial value problem $y^1 = 1 + y^2$, $y(0) = 0$ is :

(A) $y(x) = \tan x$

(B) $y(x) = x - \frac{1}{3}x^3 - \frac{2}{15}x^5 + ...$

(C) $y(x) = x + \frac{1}{3}x^2 + \frac{2}{15}x^4 + ...$

(D) $y(x) = x + \frac{2}{3}x^3 + \frac{1}{15}x^5 + ...$

51. The partial differential equation formed from the equation that represents the set of all spheres whose centre lie along the z-axis is given by :

(A) $xp - yq = 0$ (B) $yp - zq = 0$ (C) $yp - xq = 0$ (D) $xp - zq = 0$

52. The complete integral of the equation $(p^2 + q^2)x = pz$ is :

(A) $z^2 = a^2x + (ay + b)^2$ (B) $z = a^2x^2 + (ay + b)^2$

(C) $z^2 = a^2x^2 + (ay + b)$ (D) $z^2 = a^2x^2 + (ay + b)^2$

53. The integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$, which passes through the line $x_0(s) = 1$, $y_0(s) = 0$ and $z_0(s) = s$ is :

(A) $x^2 + y^2 + z^2 - xz - y + z = 1$ (B) $x^2 + y^2 - xyz - y + z = 1$

(C) $x^2 + y^2 - xz - y + z = 1$ (D) $x^2 + y^2 - xz - yz + z = 1$

54. Which of the following are solutions to the partial differential equation : $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$

(A) $\cos(3x - y)$ (B) $x^2 + y^2$ (C) $\sin(3x - 3y)$ (D) $e^{-3\pi x}\sin \pi y$

55. The partial differential equation $5\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial y^2} = xy$ is classified as :

(A) elliptic (B) parabolic (C) hyperbolic (D) none of the above

56. The finite difference approximation of $\frac{\partial^2 u}{\partial x^2}$ in the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at (x, y) can be approximated as :

(A) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x + \Delta x, y) - 2u(x,y) + u(x - \Delta x, y)}{(\Delta x)^2}$

(B) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x + \Delta x, y) - u(x,y) + u(x - \Delta x, y)}{(\Delta x)^2}$

(C) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x, y + \Delta y) - 2u(x,y) + u(x, y - \Delta y)}{(\Delta x)^2}$

(D) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x + \Delta x, y) - u(x - \Delta x, y)}{2\Delta x}$

57. Which of the following is an example of a parabolic type partial differential equation ?
(A) Wave equation
(B) Heat equation
(C) Laplace's equation
(D) None of the above

58. The following function(s) can be used for interpolation :
(A) Polynomial
(B) Exponential
(C) Trigonometric
(D) All of the above

59. Given, $\frac{d^2 y}{dx^2} = 6x - 0.5x^2$, $y(0) = 0$, $y(12) = 0$ the value of $\frac{d^2 y}{dx^2}$ at $y(4)$ using the finite difference method and a step size of $h = 4$ can be approximated by :

(A) $\frac{y(8) - y(0)}{8}$
(B) $\frac{y(8) - 2y(4) + y(0)}{16}$
(C) $\frac{y(12) - 2y(8) + y(4)}{16}$
(D) $\frac{y(4) - y(0)}{4}$

60. The following x - y data is given :

x	15	18	22
y	24	37	25

The Newton's divided difference second order polynomial for the above data is given by :
$f_2(x) = b_0 + b_1(x - 15) + b_2(x - 15)(x - 18)$
The value of b_1 is most nearly :
(A) −1.0480
(B) 0.14333
(C) 4.3333
(D) 24.000

61. Where is the minimum criterion used ?
 (A) When there is an AND operation (B) When there is an OR operation
 (C) In De Morgan's theorem (D) None of the above

62. Which of the following statements is true ?
 (A) A number is rational if and only if its square is rational.
 (B) An integer n is odd if and only if n^2+2n is odd.
 (C) A number is irrational if and only if its square is irrational.
 (D) A number n is odd if and only if $n(n+1)$ is even.

63. Consider the following language $L=\{a^n b^n c^n d^n | n \geq 1\}$ L is :
 (A) CFL but not regular (B) CSL but not CFL
 (C) regular (D) type 0 language but not type 1

64. Which of the following is a level set of $f(x_1, x_2)=x_1-x_2$
 (A) $\{(x_1, x_2)|x_1-2=x_2+1\}$ (B) $\{(x_1, x_2)|x_1{}^2=x_2\}$
 (C) $\{(x_1, x_2)|x_1-2=0\}$ (D) $\{(x_1, x_2)|x_1{}^2=x_2{}^2\}$

65. Which of the following is false ?
 (A) Product of T_1 - spaces is a T_1 - space
 (B) Product of completely regular spaces is completely regular
 (C) Product of first countable spaces is first countable
 (D) Product of two second countable spaces is second countable

66. What's the highest dimension a fractal can have ?
 (A) 1 (B) 2 (C) 3 (D) 4

67. The Lorentz Butterfly is an example of what type of fractal ?
 (A) Julia set (B) Mandelbrot set
 (C) Strange Attractor (D) None of the above

68. What integral equation is equivalent to the initial value problem $y'=t^2-y$, $y(-1)=2$.

 (A) $y(t)=-1+\int_2^t (s^2-y(s))ds$ (B) $y(t)=2+\int_{-1}^t (s^2-y(s))ds$

 (C) $y(t)=2+\int_{-1}^t sy(s)ds$ (D) None of the above

69. Who were the two mathematicians that invented calculus ?
 (A) Newton and Laplace (B) Newton and Euler
 (C) Newton and Gauss (D) Newton and Leibniz

70. For the differential equation $4x^3y''+6x^2y'+y=0$ the point at infinity is :
 (A) an ordinary point (B) a critical point
 (C) an irregular singular point (D) a regular singular point