1. Which of the following statements is **not** correct ?
   (A) The family F consisting of all finite subsets of Ω and their complements always is a σ-field.
   (B) The family F consisting of all countable subsets of Ω and their complements always is a σ-field.
   (C) If F_1 and F_2 are σ-fields on Ω, then $F_1 \cap F_2$ is also a σ-field.
   (D) If F is a σ-field on Ω and $A \subset \Omega$, then $G = \{A \cap B : B \in F\}$ is a σ-field on A.

2. Which of the following is a sufficient condition for the real valued function f to be Riemann integrable with respect to α on [a, b] ?
   (A) f is continuous on [a, b]
   (B) f is of bounded variation on [a, b]
   (C) f is of bounded variation on [a, b] and α is continuous on [a, b]
   (D) α is of bounded variation on [a, b]

3. Which of the following sequences $\{f_n\}$ is not uniformly convergent on [0, 1] ?
   (A) $f_n(x) = \dfrac{x}{1 + nx^2}$
   (B) $f_n(x) = \dfrac{\log(1+n^3x^2)}{n^2}$
   (C) $f_n(x) = x - \dfrac{x^n}{n}$
   (D) $f_n(x) = nx(1-x^2)^n$

4. Let A be a set of rational numbers in [0, 10]. Then the Lebesgue measure of A is :
   (A) 10 (B) 0 (C) ∞ (D) None of these

5. If $[x]$ denotes the greatest integer function, then the value of the Riemann-Stieltjes integral $\int_0^2 [x]\, d(x^2)$ is :
   (A) 3 (B) 8/3 (C) 16/3 (D) 9

6. The following vectors (1, 9, 9, 8), (2, 0, 0, 8), (2, 0, 0, 3) are :
   (A) Linearly dependent (B) Linearly independent
   (C) Identical vectors (D) None of these

7. The matrix $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is a :
   (A) Hermitian matrix (B) Skew Hermitian matrix
   (C) Symmetric matrix (D) Skew symmetric matrix

8. If A is a 6×6 matrix of rank 4 then its nullity is :
   (A) 2 (B) 4 (C) 6 (D) 10

9. What type of definiteness does the quadratic form $x^2+2y^2+3z^2+2yz-2xz+2xy$ possess ?
   (A) Positive definite (B) Negative definite
   (C) Positive semi - definite (D) Indefinite

10. Identify the **incorrect** statement of the following :
   (A) If A is $m \times n$ matrix of any rank with its g-inverse A^*, then $AA^* A = A$
   (B) The g - inverse A^* exists and rank $(A^*) \geq$ rank (A)
   (C) If A^* is a g - inverse then A^*A or AA^* is idempotent
   (D) The Moore - Penrose g - inverse is not unique

11. Which of the following doesn't hold true always ?
   (A) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq 1 - \sum_{i=1}^{n} P(A_i^c)$
   (B) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq -(n-1) + \sum_{i=1}^{n} P(A_i)$
   (C) $P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$
   (D) $P\left(\bigcup_{i=1}^{n} A_i\right) \leq 1 - \sum_{i=1}^{n} P(A_i^c)$

12. If $X > 0$ a.s., then which of the following inequalities is wrong ?
   (A) $E(X^2) \geq (E(X))^2$ (B) $E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$
   (C) $E(X^{1/2}) \geq (E(X))^{1/2}$ (D) $E(\log(X)) \leq \log(E(X))$

13. If X is a uniform [0, 1] random variable, then which of the following is a lower bound for $P\left\{\left|X - \frac{1}{2}\right| \leq \frac{1}{4}\right\}$?
   (A) 8/9 (B) 2/3 (C) 2/9 (D) 1/9

14. Choose the **correct** statement :
   (A) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$
   (B) $X_n \xrightarrow{a.s} X \Rightarrow X_n \xrightarrow{P} X$
   (C) $X_n \xrightarrow{L} X$, $Y_n \xrightarrow{P} c \Rightarrow X_n + Y_n \xrightarrow{P} X + c$
   (D) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{m.s.} X$

15. Let $\{X_n\}$ be a sequence of random variables with
   $P(X_n = n) = \frac{1}{n^r}$, $P(X_n = 0) = 1 - \frac{1}{n^r}$, $r > 0$. Then,
   (A) $X_n \xrightarrow{P} 0$ (B) $X_n \to 0$ in r^{th} mean
   (C) X_n converges to 1 in probability (D) None of these

16. If X is a non - negative random variable with distribution function F, then which of the following represents E(X) ?

(A) $\int_0^\infty xF(x)dx$ (B) $\int_0^\infty x\,(1-F(x))dx$

(C) $\int_0^\infty F(x)dx$ (D) $\int_0^\infty (1-F(x))dx$

17. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both contracts ?

(A) 1/5 (B) 4/9 (C) 14/45 (D) 1

18. Let $\{X_n\}$ be a sequence of i.i.d. random variable with $E(X_n^2) < \infty$. Let $S_n = \sum_{k=1}^{n} X_k$, $\sigma^2 = \text{Var}(X_n) > 0$ and $Z \sim N(0, 1)$. Then,

(A) $\frac{S_n - E(S_n)\,d}{n\sigma} \to Z$ (B) $\frac{S_n - E(S_n)\,d}{n\sigma^2} \to Z$

(C) $\frac{S_n - E(S_n)\,d}{\sqrt{n}\sigma^2} \to Z$ (D) $\frac{S_n - E(S_n)\,d}{\sqrt{n}\sigma} \to Z$

19. Which type of convergence is used in central limit theorem ?

(A) Convergence in probability (B) Convergence in r^{th} mean

(C) Convergence in Law (D) None of these

20. Which of the following is **not** a characteristic function ?

(A) $e^{-\frac{t^2}{2}}$ (B) $\frac{2}{1+\cos t}$ (C) $e^{-|t|}$ (D) $(1-2it)^{-\frac{1}{2}}$

21. If $P_x(s)$ denote the p.g.f of the random variable X, then the p.g.f of $Y = mX + n$, where m, n are integers $(m \neq 0)$ is given by :

(A) $s^n P_x(s^m)$ (B) $sP_x(s^m)$ (C) $sP_x(s)$ (D) $s^m P_x(s^n)$

22. Let X and Y be independent standard normal random variables. Then the distribution of $Z = \left(\frac{X-Y}{X+Y}\right)^2$ is :

(A) Chi - square distribution with 2 d.f

(B) Chi - square distribution with 1 d.f

(C) F distribution with (2, 2) d.f

(D) F distribution with (1, 1) d.f

23. The distribution function F of a two - dimensional random variable satisfies, for (x_1, y_1), (x_2, y_2), $x_1 < x_2$, $y_1 < y_2$ satisfies :

(A) $F(x_2, y_2) - F(x_2, y_1) + F(x_1, y_1) - F(x_1, y_2) \geq 0$

(B) $F(x_1, y_1) - F(x_2, y_2) + F(x_2, y_1) + F(x_1, y_2) \geq 0$

(C) $F(x_2, y_1) + F(x_1, y_2) - F(x_1, y_1) - F(x_2, y_2) \geq 0$

(D) $F(x_1, y_1) + F(x_2, y_1) + F(x_2, y_2) - F(x_1, y_2) \geq 0$

24. If ρ_{XY} is the correlation between X and Y, then correlation between $U = a + cX$ and $V = b - dY$, $(a, b, c, d > 0)$ is :

(A) ρ_{XY} (B) $-\rho_{XY}$ (C) $\frac{cd}{ab}\rho_{XY}$ (D) $\frac{ab}{cd}\rho_{XY}$

25. In a partially destroyed laboratory record of an analysis of correlation data, only the following regression equations are legible :

$8x - 10y + 66 = 0$

$40x - 18y - 214 = 0$

Then the mean value of Y is :

(A) 13 (B) 17 (C) 18 (D) 66

26. If $X \sim$ Poisson (λ_1), $Y \sim$ Poisson(λ_2) and X and Y are independent, then the conditional distribution of X given $X + Y$ is :

(A) Binomial (B) Poisson

(C) Negative binomial (D) None of these

27. If X and Y are random variables with distribution functions $F(x) = 1 - e^{-x}$ and $G(y) = 1 - e^{-2y}$ respectively, then $E[F(Y) + 2g(X)]$ is :

(A) 0 (B) 1 (C) 3/2 (D) ∞

28. The distribution of $X_{(n)} = \text{Max.}(X_1, X_2, \ldots\ldots X_n)$, where $X_1, X_2, \ldots\ldots X_n$ are i.i.d. $\beta(\alpha, 1)$ is :

(A) $\beta(\alpha n, 1)$ (B) $\beta(\alpha, 1)$ (C) $\beta(\alpha, n)$ (D) $\beta(\alpha n, n)$

29. If X follows Pareto distribution, then the distribution of $Y = \frac{1}{X}$ is :

(A) Cauchy (B) Pareto (C) Uniform (D) Weibull

30. If the moment generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, then its mean and variance are respectively :

(A) 2 and 0.5 (B) 1 and 0.8 (C) 2 and 0.8 (D) 1 and 0.5

31. Which of the following statements is **not correct** ?
(A) If a sample of size n is drawn from a population of size N in such a way that every possible sample of size 'n' has the same chance of being selected, the sample thus obtained is a simple random sample
(B) Simple random sample doesn't attempt to reduce the effect of data variation on the error of estimation
(C) A stratified random sample is one obtained by separating the population elements into non-overlapping groups and then selecting a simple random sample from each group
(D) Stratified random sample always produces an estimator with a smaller variance than that of the corresponding estimator in simple random sampling

32. With regard to ratio estimation based on the response variable y and subsidiary variable x, identify the **correct** statement of the following :
(A) The use of ratio estimator is most effective when the relationship between y and x is linear through the origin and Var(y) is proportional to x
(B) The correlation ρ between y and x does not affect the precision of the ratio estimator
(C) Ratio estimators are always unbiased
(D) The ratio estimation is usually superior to regression in estimation if the relationship between the y' s and x's is a straight line not through the origin

33. A sociologist wants to estimate the average income per adult male in a certain small city where no list of resident adult is available. What would be the logical choice of the survey design in this case ?
(A) Simple random sampling (B) Systematic sampling
(C) Stratified sampling (D) Cluster sampling

34. Which of the following statements is **not correct** ?
(A) Systematic sampling is easier to perform and is less subject to interviewers errors than simple random sampling
(B) Systematic sampling often provides more information per unit cost than simple random sampling
(C) When the population size (N) is large and the correlation coefficient (ρ) between pairs of elements within the systematic sampling is closer to one, then the systematic sampling may be better than simple random sampling
(D) A systematic sampling is preferable when the population of interest is ordered and population size is large

35. A national survey of University student opinions is conducted by selecting a simple random sample of universities from all those in the country and then selecting a simple random sample of students from each university. What sort of sample survey design is used here ?
(A) Stratified random sampling (B) A two - stage cluster sampling
(C) Cluster sampling (D) None of these

36. In a BIBD, four treatments are arranged in four blocks of three plots each. Each treatment occurs once and only once in three blocks and any two treatments occur together in λ blocks. Then the value of λ is :
(A) 1 (B) 2 (C) 3 (D) 4

37. Under a 2^4-factorial design, what interaction is confounded with the following blocks ?

Block 1 :	(1)	ad	ac	ab	cd	bd	bc	abcd
Block 2 :	a	d	c	b	acd	abd	abc	bcd

(A) AB (B) AC (C) ABC (D) ABCD

38. Consider the linear model :

$y_1 = \theta_1 + 2\theta_2 - 2\theta_3 + \epsilon_1$

$y_2 = \theta_1 + 3\theta_2 - \theta_3 + \epsilon_2$

$y_3 = \theta_2 + \theta_3 + \epsilon_3$

Where y_i are observations, θ_i are parameters and ϵ_i are uncorrelated random variables with mean zero and constant variance for i = 1, 2, 3. Then which of the following is **true** ?

(A) $2y_1 - y_2 - y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$

(B) $2y_1 - y_2 - y_3$ is the BLUE of $\theta_1 - 4\theta_3$

(C) $y_2 - 3y_3$ is the BLUE of $\theta_1 - 4\theta_3$

(D) $y_1 - 4y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$

39. In the ANOVA for an RBD with 4 blocks and 6 treatments having one missing value which is estimated, the error degrees of freedom is :

(A) 14 (B) 15 (C) 23 (D) 24

40. For which of the following set of values will a Balanced Incomplete Block Design of parameters (v, b, r, k, λ) exist ?

(A) v = 11, b = 22, r = 6, k = 3, λ = 1 (B) v = 7, b = 7, r = 4, k = 4, λ = 2

(C) v = 21, b = 4, r = 4, k = 21, λ = 4 (D) v = 7, b = 6, r = 3, k = 3, λ = 1

41. Test statistic for test the significance of difference between variance of two normal populations with known means based on two independent samples of sizes m and n respectively follows :

(A) an F distribution with $(m-1, n-1)$ d.f.

(B) a t distribution with $(m+n)$ d.f.

(C) a t distribution with $(m+n-2)$ d.f.

(D) an F distribution with (m, n) d.f.

42. If $X_1, X_2, \ldots\ldots X_n$ is a random sample from a normal distribution $N(\mu, \sigma^2)$ then the MLE of σ^2 when μ is known is :

(A) Unbiased and consistent (B) Unbiased but not consistent

(C) Consistent but not unbiased (D) Neither consistent nor unbiased

43. If $X_1, X_2, \ldots\ldots X_n$ is a random sample from a population with density function $f(x) = \frac{1}{2}e^{-|x-\theta|}, -\infty < x < \infty$, then the MLE of the parameter θ is :

(A) Max $(X_1, X_2, \ldots\ldots X_n)$ (B) Mean of $X_1, X_2, \ldots\ldots X_n$

(C) Min $(X_1, X_2, \ldots\ldots X_n)$ (D) Median of $X_1, X_2, \ldots\ldots X_n$

44. A test with size α and power β is said to be an unbiased test if :

(A) $\alpha \geq \beta$ (B) $\alpha \leq 1-\beta$ (C) $\alpha \leq \beta$ (D) $\alpha \geq 1-\beta$

45. If $X_1, X_2, \ldots\ldots, X_n$ be a random sample from N (θ, θ^2) then which of the following is **true** :

(A) ΣX_i is sufficient for θ

(B) ΣX_i^2 is sufficient for θ

(C) $(\Sigma X_i, \Sigma X_i^2)$ is jointly sufficient for θ

(D) $\left(\Sigma X_i, \Sigma(X_i - \overline{X})^2\right.$ is jointly sufficient for θ

46. For an SPRT with strength (α, β) the boundary points A and B satisfy :

(A) $A \leq \frac{\beta}{1-\alpha}, B \geq \frac{1-\beta}{\alpha}$ (B) $A \leq \frac{1-\beta}{\alpha}, B \geq \frac{\beta}{1-\alpha}$

(C) $A \leq \frac{\alpha}{1-\beta}, B \geq \frac{\beta}{1-\alpha}$ (D) None of these

47. Based on a single observation X from a normal population with mean μ and variance unity the UMP critical region of size α for test the hypothesis $H_0 : \mu \geq 2$ against $H_1 : \mu < 2$ when z_α is the α^{th} quantile of a standard normal distribution is :

(A) $X < z_\alpha - 2$ (B) $X > z_\alpha - 2$ (C) $X < z_\alpha + 2$ (D) $X > z_\alpha + 2$

48. Based on a random sample of size n from B (1, p), the UMVUE of $p(1-p)$ is :

(A) $\frac{\Sigma X_i (n - \Sigma X_i)}{n-1}$ (B) $\frac{\Sigma X_i (\Sigma X_i - n)}{n-1}$

(C) $\frac{\Sigma X_i (n - \Sigma X_i)}{n(n-1)}$ (D) $\frac{\Sigma X_i (\Sigma X_i - n)}{n(n-1)}$

49. Let T be an ancilliary statistic for the parameter θ. Then T and S are independent if :

(A) S is complete sufficient for θ (B) S is unbiased for θ

(C) S is sufficient for θ (D) S is consistent for θ

50. If W is the Wilcoxon rank sum statistic and U is the Mann - Whiteny U statistic, then :

(A) $U = W + \frac{n(n+1)}{2}$ (B) $U = W - \frac{n(n+1)}{2}$

(C) $W = U + \frac{n(n+1)}{2}$ (D) None of these

51. If X follows $N_p (0, \Sigma)$ then $X'AX$ follows chi-square distribution with r d.f if and only if :

(A) $A\Sigma$ is of rank r (B) A is of rank r

(C) $A\Sigma$ is idempotend of rank r (D) A is idempotend of rank r

52. Let R be the sample correlation coefficient of a bivariate sample of size n from a bivaraite normal population with correlation coefficient ρ. If $\rho = 0$ then the statistic $R\sqrt{\frac{n-2}{1-R^2}}$ follows a :

(A) t distribution with $(n-1)$ d.f. (B) t distribution with $(n-2)$ d.f.
(C) F distribution with (1, n) d.f. (D) F distribution with $(1, n-1)$ d.f.

53. The distribution of Hotelling's T^2 statistic follows :

(A) Chi - square distribution (B) t distribution
(C) Wishart distribution (D) F distribution

54. Let $X_1, X_2, \ldots\ldots X_n$ $(n > P)$ be iid random vectors from $N_p(\theta, \Sigma)$. Then the distribution of $S = \frac{1}{n}\Sigma(X_i - \overline{X})(X_i - \overline{X})'$, Where $\overline{X} = \frac{1}{n}\Sigma x_1$ is :

(A) $W_p(\Sigma, n-1)$ (B) $W_p\left(\frac{\Sigma}{n}, n\right)$ (C) $W_p\left(\frac{\Sigma}{n}, n-1\right)$ (D) $W_p(\Sigma, n)$

55. If $(X_1, X_2)'$ follows a bivaraite normal distribution with dispersion matrix $\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ then the dispersion matrix of $(X_1 + X_2, X_1 - X_2)'$ is :

(A) $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

56. If $\{N_t, t \in T\}$ is a Poisson process with rate λ then which of the following is/are **correct** :

(a) N_t has independent increment
(b) $P(N_{t+h} - N_t = 1) = \lambda t + 0(h)$
(c) N_t is a stationary increment process

(A) a, b and c (B) a and b (C) a and c (D) b and c

57. If $\{X_n, n = 0, 1, \ldots\ldots\}$ is a Galton - Watson branching process with $E(X_1) = m$, then the probability of ultimate extinction is 1 if :

(A) $m > 1$ (B) $m > 2$ (C) $m \leq 2$ (D) $m \leq 1$

58. Let Y_1, Y_2 and Y_3 be the three principal components of the random vector $X = (X_1, X_2, X_3)$. Which of the following is/are **true** :

(a) $V(Y_1) + V(Y_2) + V(Y_3) = V(X_1) + V(X_2) + V(X_3)$
(b) $V(Y_1) \geq V(Y_2) \geq V(Y_3)$
(c) Y_1, Y_2 and Y_3 are uncorrelated

(A) (a) and (c) (B) (b) and (c) (C) (a) and (b) (D) (a), (b) and (c)

59. Which of the following statement(s) is/are **true** ?

(a) The state space of a finite Markov chain contain atleast one recurrent state
(b) In a finite irreducible Markov chain all states are non-null recurrent
(c) A null recurrent aperodic state of a Markov chain is called ergodic

(A) (a) and (b) (B) (a) and (c) (C) (b) and (c) (D) (a), (b) and (c)