

Syllabus for Mathematics - Assistant Professor Examination -Collegiate Education

Module I - Linear Algebra:

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem.

Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms-rational forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic form.

Module II - Real Analysis:

Sequences and series, convergence, $\lim \sup$. $\lim \inf$. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, Rolle's theorem, Mean value theorem. Sequences and series of functions-uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Double and triple integrals,

Module III - Real Analysis(continued):

Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, inverse and implicit function theorems. Special functions- Beta and Gamma functions, Fourier series.

Module IV - Abstract Algebra:

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, isomorphisms, cyclic groups, permutation groups, Cayley's theorem, Direct products, Fundamental theorem for abelian groups, class equations, Sylow theorems.

Module V - Abstract Algebra (continued):

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

Module VI - Topology:

Metric spaces, continuity, Topological spaces, Base, subbase, countability properties, Separation axioms, Compact space, one point compactification, locally compact space, connected spaces, pathwise connectedness, Quotient spaces, Product topology.

Module VII - Complex Analysis:

Complex numbers, polar form, properties of complex numbers, Analytic functions, Cauchy Reimann equations, Conformal Mappings, Mobius transformation, Power series, Zeros of analytic functions, Liouville's theorem, Complex integration, real integrals using complex integration, Cauchy's theorem and Cauchy's integral formula, Morera's theorem, open mapping theorem, Singularities and its classification, residues, Laurent series, Schwarz lemma, Maximum modulus principle, Argument principle.

Module VIII - Functional Analysis:

Normed Linear spaces, Continuity of linear maps, Banach spaces, Hahn Banach spaces, Open mapping theorem, closed graph theorem, uniform boundedness principle, Inner product spaces, Hilbert spaces Projections, Bounded operators, Normal, unitary and self adjoint operators.

Module IX - Ordinary Differential & Partial Equations :

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs.

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Module X - Theory of Numbers:

Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , congruences, Chinese Remainder Theorem, Euler's ϕ -function, Fermat's theorem, Wilson's theorem, Euler's theorem, primitive roots.