129

064/21

Total Number of Questions : 27

Time : 2.00 Hours

Max. Marks : 100

1. A point is chosen at random inside a circle. Find the probability that the point is more close to the centre of the circle than the circumference of the circle. **(3 Marks)**
2. Show that Binomial distribution belongs to the exponential family of distributions. **(3 Marks)**
3. In order to test whether a coin is perfect, it is tossed 5 times. The null hypothesis of perfectness is rejected if and only if more than 4 heads are obtained. Find the probability of type I error. **(3 Marks)**
4. Give the layouts of a 4×4 Latin square design with the treatments A, B, C and D. **(3 Marks)**
5. Define Binomial distribution. Find its mean and variance. **(3 Marks)**
6. Examine whether $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f. **(3 Marks)**
7. Define Hotelling's T^2-statistics. **(3 Marks)**
8. What is the difference between random sampling and non-random sampling ? **(3 Marks)**
9. S. T. sample variance is an unbiased estimate of the population variance. **(3 Marks)**
10. Explain the problem of auto-correlation in general regression model. **(3 Marks)**
11. Define quadratic form. **(3 Marks)**
12. Define the following : **(3 Marks)**
    i) Matrix
    ii) Idempotent matrix
    iii) Rank of a matrix.
13. Define transition probability matrix. **(3 Marks)**
14. State Ergodic theorem. **(3 Marks)**
15. If X is a random variable with mean α, show that the value of $E(X-t)^2$ is minimum when $t = \alpha$. **(4 Marks)**
16. Let $X_1, X_2, \ldots X_n$ be a random sample from a population having uniform distribution in the interval $\left(\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma\right)$. Find the estimators of μ and σ by the method of moments. **(4 Marks)**
17. Given H_0 : Median = 5. Compute T^+, T^- and T for the following observations :
    6, 10, 3, 5, 2, 12 **(4 Marks)**
18. Define Weibull distribution. Find its mean and variance. **(4 Marks)**

P.T.O.

19. Define χ^2, t and F distributions. **(4 Marks)**

20. Describe spectral decomposition of a symmetric matrix. **(4 Marks)**

21. State Cayley – Hamilton theorem. Also using Cayley – Hamilton theorem, compute A^{-1} in terms of powers of A. **(4 Marks)**

22. Let $\overline{X}_n$ be the mean of a random sample of size n from a population with mean μ and variance σ^2. Show that $\overline{X}_n$ converges stochastically to μ, if σ^2 is finite. **(5 Marks)**

23. Derive the confidence interval for the difference of means of two normal populations, based on small samples of sizes n_1 and n_2 taken from the populations by assuming the variances of the populations are same. **(5 Marks)**

24. Let X have the distribution $f(x, \lambda) = \lambda^x(1 - \lambda)^{1-x}$; $x = 0, 1; 0 < \lambda < 1$. Construct the SPRT for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$. **(5 Marks)**

25. For a resolvable BIBD with parameters v, b, r, k and λ, show that $b \geq v + r - 1$. **(5 Marks)**

26. a) Define joint, marginal and conditional p.d.f. of multivariate distributions.
    b) What is lack of memory property ? Give discrete and continuous distributions having lack of memory property.
    c) Define multivariate normal distribution. **(5 Marks)**

27. In the general linear model $\underline{Y} = X\underline{\beta} + \underline{U}$, with usual notations and if $\underline{U}$ follows $N_n\left(\underline{0}, \sigma^2, I_n\right)$, derive an unbiased estimator for σ^2. **(5 Marks)**
