1083

058/21

Total Number of Questions : 32

Time : 3.00 Hours   Max. Marks : 200

1. Expand $e^{x+\left(\frac{1}{2}\right)}$ in powers of $x-1$. **(2 Marks)**
2. Differentiate $\int_1^{x^3} \cos t\,dt$ with respect to 'x'. **(2 Marks)**
3. If the normals at two points of the parabola $y^2 = 4x$ intersect on the curve, find the product of the ordinates of the two points. **(2 Marks)**
4. A straight line and a conic are described in polar forms as $\frac{l}{r} = 3\cos\theta + \sin\theta$ and $\frac{l}{r} = 1 + e\cos\theta$ respectively. If the line touches the conic at some point, find the eccentricity 'e' and identify the conic. **(2 Marks)**
5. Find the values of log (– 1) and log (– 1). **(2 Marks)**
6. A particle moving along the curve C has an instantaneous velocity $8 - \text{cosec}^2 t$. Obtain the path C described by the particle, given that it passes through the point $\left(\frac{\pi}{4}, 0\right)$. **(4 Marks)**
7. Compute the area between the curve $y = \sin 2x$ and the x-axis from $x = 0$ to $x = 2\pi$. **(4 Marks)**
8. Evaluate $\int_0^{\sqrt{\pi}} \int_{x^2}^{\pi} \frac{\sin y}{\sqrt{y}}\,dy\,dx$. **(4 Marks)**
9. Find the eccentricity of the ellipse whose one pair of conjugate diameters are $y = x + 3$ and $3y + 2x + 5 = 0$. **(4 Marks)**
10. Identify the points on the region $R : 0 \le x \le \pi, 0 \le y \le 1$, where the complex function $f(z) = \sin z$ has a maximum value. **(4 Marks)**
11. Determine the range and kernel of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$. **(5 Marks)**
12. Determine the volume of the cone cut from the unit solid sphere by the cone $\varphi = \frac{\pi}{3}$, where (ρ, φ, θ) is any point in space in spherical coordinates. **(5 Marks)**
13. Find the partial differential equation satisfied by the set of all spheres of radius 'r' with their centers on the xy-plane. **(5 Marks)**
14. Construct all the distinct possible composition tables for the group (G, ∗), where G = {e, a, b, c}, 'e' being the identity element for the binary composition '∗'. **(5 Marks)**
15. Prove that all the values of i^{-4i} are real. **(5 Marks)**
16. Let G be a positively oriented simple closed contour in the complex plane and 'z' is a point inside C. Find the value of $g(z) = \int_C \frac{s^3 + 2s}{(s-z)^3}\,ds$. **(5 Marks)**
17. Prove that the function $f(x) = \sin x$ is uniformly continuous on $[0, \infty)$. **(5 Marks)**
18. Sum the series : $1 + \frac{1}{2}\cos 2\theta - \frac{1}{2.4}\cos 4\theta + \frac{1.3}{2.4.6}\cos 6\theta - \ldots$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. **(7 Marks)**

19. Let S_2 denote the set of all symmetric matrices in $M(2, \mathbb{R})$, the set of all 2×2 matrices with entries as real numbers. Show that S_2 is a vector subspace of $M(2, \mathbb{R})$ over $\mathbb{R}$. Determine the dimension of S_2 as a vector space over $\mathbb{R}$. **(7 Marks)**

20. Show that the real part of $f(z) = \frac{i}{z^2}$ is harmonic in the xy-plane that doesn't contain the origin. **(7 Marks)**

21. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ using Laurent series in the domains $D_1 : 1 < |z| < 2$ and also in $D_2 : |z| > 2$. **(7 Marks)**

22. Let 'z' be a complex variable and $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Show that Cauchy-Riemann equations are satisfied at (0, 0), but the function is not differentiable at (0, 0). **(7 Marks)**

23. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (x+y)i + (2x-z)j + (y+z)k$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(10 Marks)**

24. Determine whether the vector field $\vec{F} = (\ln x + \sec^2(x+y))\,i + \left(\sec^2(x+y) + \frac{y}{y^2+z^2}\right) j + \left(\frac{y}{y^2+z^2}\right) k$ is conservative and find a potential function for it. **(10 Marks)**

25. Let 'ρ' be the permutation $\begin{pmatrix} a & b & c & d & e \\ b & d & e & a & e \end{pmatrix}$. Find the cyclic group generated by 'ρ' with permutation multiplication as composition. Also determine the inverses of the elements of this cyclic group. **(10 Marks)**

26. Describe the group $\mathbb{Z}_{18}$. Determine all the subgroups of $\mathbb{Z}_{18}$ and draw the subgroup diagram. **(10 Marks)**

27. Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$. **(10 Marks)**

28. Evaluate $\int_1^3 (2x+1)\,dx$ as the limit of sums using a partition of [1, 3]. **(10 Marks)**

29. Evaluate $\int_C \frac{\cosh \pi z}{z(z^2+1)}\,dz$, where C is the circle in the z-plane $|z| = 2$ described in the anticlockwise direction. **(10 Marks)**

30. Solve the differential equation : **(10 Marks)**
$$x^3y''' + x^2y'' - 2xy' + 2y = 0$$

31. Find two linearly independent series solutions in powers of 'x' of the equation $y'' - 2xy' + 2py = 0$, where 'p' is a constant. **(10 Marks)**

32. Find the Cauchy's Principal Value of the integral : **(10 Marks)**
$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 2x + 2}\,dx$$