

PROVISIONAL ANSWER KEY

Question 21/2023/OL

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Question1:-If f is convex and EX is finite, then $f(EX) \leq Ef(X)$ is known as

A:-Liapounov's inequality

B:-Schwarz inequality

C:-Jensen's inequality

D:-Cauchy inequality

Correct Answer:- Option-C

Question2:-If X and Y are uniformly distributed on $(0, 1)$, then $E(|X - Y|)$ is

A:- $\geq \frac{1}{2}$

B:- $\leq \frac{1}{2}$

C:- ≥ 1

D:- ≤ 0

Correct Answer:- Option-A

Question3:-Let X be a random variable with characteristic function given by $\Phi(u)$, then the value of $\text{Var}(\text{Sin } x) + \text{Var}(\text{Cos } x) =$

A:- $1 - |\Phi(0)|^2$

B:- $|1 - \Phi(0)|^2$

C:- $1 - |\Phi(1)|^2$

D:- $|1 - \Phi(1)|^2$

Correct Answer:- Option-C

Question4:-If probability of hitting a target is 0.5, what is the probability of hitting the target on 6^{th} attempt

A:-0.5

B:- $(0.5)^6$

C:-0.25

D:- $(0.5)^5$

Correct Answer:- Option-B

Question5:-Buses travel on every half hours in day times in a remote area. What is the probability that a man reaching bus stop in day time will have to wait for 20 minutes ?

A:-0.5

B:-0.25

C:-0.167

D:-0.33

Correct Answer:- Option-D

Question6:-What is the probability of success in an event with both mean and variance are found 4, assuming binomial distribution ?

A:-0

B:-1

C:- $0 < p < 1$

D:-0.5

Correct Answer:- Option-A

Question7:-The points of inflexion of a normal curve are

A:- $\mu \pm \sigma$

B:- $\mu \pm 2\sigma$

C:- $\mu \pm 3\sigma$

D:- $\mu \pm 0.5$

Correct Answer:- Option-A

Question8:-The characteristic function for Cauchy distribution is

A:- $e^{-\frac{t^2}{2}}$

B:- $e^{-|t|}$

C:- $\frac{1}{1+t^2}$

D:- $\frac{1}{\pi(1+t^2)}$

Correct Answer:- Option-B

Question9:- $X_i = 1$, if the i^{th} outcome is a success with $p(\text{success}) = p$

$X_i = 0$, if the i^{th} outcome is a failure with $p(\text{failure}) = q$.

Then the distribution of random variables $S_n = \sum X_i$ where X_i are independent is _____ as n tends to infinity.

A:-Uniform

B:-Binomial

C:-Asymptotically normal

D:-Exponential

Correct Answer:- Option-C

Question10:-A fair die is thrown 600 times find the lower bound for probability of getting the number of sixes in between 80 and 120 assuming binomial distribution.

A:-0.5

B:-0.167

C:-5/6

D:-19/24

Correct Answer:- Option-D

Question11:-If an unbiased estimator and a sufficient statistic exist for T, then the minimum variance estimator for T is always a function of

A:-Unbiased estimator

B:-Sufficient Statistic

C:-Sum of both

D:-Difference of both

Correct Answer:- Option-B

Question12:-If x_1, x_2, \dots, x_n are independent observations from a normal population such that $E(x_i) = \mu$ and $V(x_i) = \sigma^2$, for $i = 1, 2, \dots, n$. Then _____ is a least square estimate of σ^2 .

A:- $\sum_{i=1}^n (x_i - \bar{x})^2$

B:- $\sum_{i=1}^n (x_i - \bar{x})$

C:- $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$

D:- $\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

Correct Answer:- Option-D

Question13:-If T_n is a sequence of estimates such that $E(T_n) \rightarrow \theta$ and $\text{Var}(T_n) \rightarrow 0$ as $n \rightarrow \infty$, then T_n is

A:-a constant

B:-1

C:-a function of θ

D:-0

Correct Answer:- Option-A

Question14:-An estimator is said to be sufficient statistic for a parametric function $T(\theta)$ if it contain all the information that is in the

A:-Unbiased estimator

B:-Sample

C:-Population

D:-Universe

Correct Answer:- Option-B

Question15:-_____ do not always exist.

A:-Unbiased estimator

B:-Sufficient Statistic

C:-Consistent estimator

D:-Efficient estimator

Correct Answer:- Option-A

Question16:-The graphical shape of t-distribution is based on

A:-sample size

B:-degrees of freedom

C:-population size

D:-error

Correct Answer:- Option-B

Question17:-The MLE of a parameter α of a population having density $\alpha - x$, $0 < x < \alpha$ is

A:- $2x$

B:- x

C:- $2/x$

D:- $x/2$

Correct Answer:- Option-A

Question18:-The method of moments were first invented by

A:-Neyman

B:-Karl Pearson

C:-Tchebyshev

D:-Fisher

Correct Answer:- Option-B

Question19:-Difference between expected value of an estimator and the corresponding parameter is

A:-mean deviation

B:-MAPE

C:-MAE

D:-bias

Correct Answer:- Option-D

Question20:-The consistency of estimators is identified by comparing

A:-Variance

B:-Mean

C:-Standard error

D:-Median

Correct Answer:- Option-A

Question21:-To test the hypothesis involving proportions, both np and nq should be

- A:-greater than 5
- B:-between 0 to 1
- C:-less than 1
- D:-greater than 50

Correct Answer:- Option-A

Question22:-Power of the test is the probability of

- A:-acceptance
- B:-correct decision
- C:-minimum error
- D:-maximum error

Correct Answer:- Option-B

Question23:-A critical region which is most powerful when a hypothesis is tested against a series of alternatives is called

- A:-Neymann CR
- B:-BCR
- C:-UMPCR
- D:-Acceptance Region

Correct Answer:- Option-C

Question24:-Neyman-Pearson Lemma is used to find _____ for testing simple H_0 against a simple H_1 .

- A:-Error
- B:-Best Critical region
- C:-Power
- D:-Significance level

Correct Answer:- Option-B

Question25:-Degrees of freedom for a chi square test of independence with contingency table of order $m \times n$ is

- A:- $mn - 1$
- B:- $n(m - 1)$
- C:- $m(n - 1)$
- D:- $(m - 1)(n - 1)$

Correct Answer:- Option-D

Question26:-Kruskal Wallis test is a nonparametric analogue of

- A:-t-test
- B:-Z-test
- C:-One-way ANOVA

D:-Two-way ANOVA

Correct Answer:- Option-C

Question27:-As the value of Chi-square near to zero in testing goodness of fit, we say that

A:-independent

B:-dependent

C:-good fit

D:-bad fit

Correct Answer:- Option-C

Question28:-Statistical hypothesis under test are called

A:-Rejection space

B:-Alternate hypothesis

C:-Composite hypothesis

D:-Null hypothesis

Correct Answer:- Option-D

Question29:-Which of the following is not a non parametric test ?

A:-F

B:-Kolmogrov-Smirnov

C:-Run's test

D:-Mann Whitney U test

Correct Answer:- Option-A

Question30:-A statistical test to determine whether there is non random association two categorical variables is

A:-Sign test

B:-Fisher's exact test

C:-Wilcoxon Signed Rank Test

D:-Friedman ANOVA

Correct Answer:- Option-B

Question31:-In an RCBD with 5 treatments each replicated in 4 blocks, the d.f. for error is

A:-16

B:-15

C:-12

D:-19

Correct Answer:- Option-C

Question32:-Analysing a CRD, if the F value in ANOVA Table is less than one, what is shows

A:-Design is wrong

B:-Some of the means are significantly different

C:-All the treatments are significantly different

D:-Analysis is wrong as higher value should be in the numerator in calculating

F

Correct Answer:- Option-A

Question33:-If we want to identify the best nutritious food for weigh gain in chicken after trying three such foods and measured weekly for one year, which analysis you prefer ?

A:-ANOVA

B:-MANOVA

C:-Repeated Measures ANOVA

D:-t

Correct Answer:- Option-C

Question34:-Replication is used in Design of Experiments to

A:-Make observation independent

B:-Make treatments independent

C:-To convince scientific world

D:-To estimate the error

Correct Answer:- Option-D

Question35:-In a uniformity trial to estimate the fertility gradient of an experimental material which method is suitable ?

A:-Pearson's Correlation

B:-Simple random sampling

C:-Fairfield Smith's Variance law

D:-Minimum curvature method by Fisher

Correct Answer:- Option-C

Question36:-When standard deviation is proportional to mean, the transformation used before doing ANOVA is

A:-Square root

B:-Arc Sine

C:-Logarithmic

D:-Inverse

Correct Answer:- Option-C

Question37:-The method used to split the factorial experiment in to two in the case of large number of treatments is

A:-Defining contrast

B:-Split plot design

C:-Strip plot design

D:-Galois group blocking

Correct Answer:- Option-A

Question38:-A symmetric BIBD exists for even value of v , where v is the number of treatments, r the number of replications and λ is the no. of pairs, only if $r-\lambda$ is a

A:-prime number

B:-a perfect square

C:-a perfect cube

D:-even number

Correct Answer:- Option-B

Question39:-In a resolvable BIBD with number of treatments v , number of replications r , the number of blocks b should be

A:- $v + r - 1$

B:- $< v + r - 1$

C:- $\geq v + r - 1$

D:- v

Correct Answer:- Option-A

Question40:-In a breeding trial with large number of lines, where RBD or BIBD can't be considered

A:-Lattice Design

B:-Augmented Design

C:-Response surface design

D:-LSD

Correct Answer:- Option-A

Question41:-In the case of study of estimation of milk production from cows in Kerala, which type of sampling design is better with limited resources and time ?

A:-Time series data collection

B:-Simple Random Sampling

C:-Cluster Sampling

D:-Stratified Random Sampling with breed

Correct Answer:- Option-C

Question42:-Unit of sample selected is known as

A:-Sampling unit

B:-Sample unit

C:-Population

D:-Standard error

Correct Answer:- Option-B

Question43:-Proximity to the value obtained by the repeated application of the sampling procedure is

A:-accuracy

B:-partition

C:-standard error

D:-precision

Correct Answer:- Option-D

Question44:-Ratio estimator is more efficient when

A:- $\rho > \frac{1c_x}{2c_y}$

B:- $\rho < \frac{1c_x}{2c_y}$

C:- $\rho > \frac{2c_x}{c_y}$

D:- $\rho < \frac{2c_x}{c_y}$

Correct Answer:- Option-A

Question45:-If there is perfect correlation with y and x so that $y = a + bx$, then in SRS the estimator \bar{y} will be superior to ratio estimator $\hat{X}\hat{R}$ if

A:-
$$\frac{\dot{x}^2 v\left(\frac{1}{\hat{x}^3}\right)}{s_x^2} = \frac{1-f}{n}$$

B:-
$$\frac{\dot{x}^2 v\left(\frac{1}{\hat{x}^3}\right)}{s_x^2} = \frac{b^2}{a^2} \frac{1-f}{n}$$

C:-
$$\frac{\dot{x}^2 v\left(\frac{1}{\hat{x}^3}\right)}{s_x^2} < \frac{b^2}{a^2} \frac{1-f}{n}$$

D:-
$$\frac{\dot{x}^2 v\left(\frac{1}{\hat{x}^3}\right)}{s_x^2} > \frac{b^2}{a^2} \frac{1-f}{n}$$

Correct Answer:- Option-D

Question46:-Intra cluster correlation coefficient ρ is calculated using the formula

A:-
$$\frac{1}{M-1} \frac{\sigma_b^2 - \sigma_w^2}{\sigma^2}$$

B:-
$$\frac{1}{M} \frac{\sigma_b^2 - \sigma_w^2}{\sigma^2}$$

C:-
$$\frac{1}{M-1} \frac{\sigma_b^2 - \sigma^2}{\sigma_w^2}$$

D:-
$$\frac{1}{M} \frac{\sigma_b^2 - \sigma^2}{\sigma_w^2}$$

Correct Answer:- Option-A

Question47:-If 'c' denote the cost of collecting information on x and c for the cost of collecting information on y, double sampling is better than direct sampling for y in

the case of regression estimation, then the condition for same cost is

A:- $\rho^2 < \frac{4cc'}{(c + c')^2}$

B:- $\rho^2 > \frac{4cc'}{(c + c')^2}$

C:- $\rho^2 > \frac{4cc'}{(c - c')^2}$

D:- $\rho^2 < \frac{4cc'}{(c - c')^2}$

Correct Answer:- Option-B

Question48:-In SRSWOR sample mean square is an unbiased estimate of population

A:-Variance

B:-Mean square

C:-Mean

D:-Standard error

Correct Answer:- Option-B

Question49:-Sampling Variance _____ due to post stratification.

A:-increases

B:-remains same

C:-decreases

D:-halves

Correct Answer:- Option-A

Question50:-In medical practices, there are some diseases which are yet to be under research, person having such diseases can be identified by some references. Such type of sampling is known as

A:-Gibbs Sampling

B:-Convenience Sampling

C:-Judgement Sampling

D:-Snowball Sampling

Correct Answer:- Option-D

Question51:-Let X be a random variable with pdf $f(x) = kx^3 e^{-2x}$ $X > 0$. The value of K is

A:- $\frac{1}{2}$

B:- $\frac{8}{3}$

C:- $\frac{1}{4}$

D:- $\frac{16}{3}$

Correct Answer:- Option-B

Question52:-Let X and Y be two independent geometric random variables. The

conditional distribution of $X|X + Y$ is

A:-Uniform

B:-Geometric

C:-Binomial

D:-Hyper Geometric

Correct Answer:- Option-A

Question53:-Consider a random experiment with two possible outcomes success and failure. Experiment is repeating n times independently. Let X denote the no. of success with constant probability p . Distribution of the no. of failure is

A:-Poisson

B:-Multinomial

C:-Geometric

D:-Binomial

Correct Answer:- Option-D

Question54:-Which of the following statements is/are correct about Cauchy distribution ?

i. For a Cauchy random variable, central moments exist

ii. For a Cauchy random variable, mean does not exist

iii. For a Cauchy random variable, characteristic function exists

A:-All of the above (i, ii and iii)

B:-Only (i and ii)

C:-Only (ii and iii)

D:-Only (i and iii)

Correct Answer:- Option-C

Question55:-Let X and Y are two independent exponential random variables with parameters 2 and 3. Which of the following is correct ?

A:- $X + Y \sim$ exponential (5)

B:- $X + Y \sim$ Gamma (5, 2)

C:- $\text{Min}(X, Y) \sim$ exponential (5)

D:- $\text{Max}(X, Y) \sim$ Exponential (5)

Correct Answer:- Option-C

Question56:-Let X_1, X_2, \dots, X_n be a set of n independent observation taken from $U(0, 2)$. Let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ be the ordered pair of sample observations. Then the joint distribution of is $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ is

A:- $n!$

B:- $\frac{1}{2^n}$

C:- $n! \left(\frac{x}{2}\right)^n$

D:- $\frac{n!}{2^n}$

Correct Answer:- Option-D

Question57:-Ratio of squares of two independent standard normal variate is

A:-t distribution with 1 degrees of freedom

B:-F distribution with (1, 1) degrees of freedom

C:-Standard Normal distribution

D:- χ^2 distribution with 1 degree of freedom

Correct Answer:- Option-B

Question58:-Suppose x_1 , x_2 and x_3 are three independent standard normal variates.

Then the distribution of $\frac{x_1 + x_2 + x_3}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$ is

A:-N (0, 3)

B:- χ^2 distribution with 3 degrees of freedom

C:-F distribution with (1, 1) degrees of freedom

D:-t distribution with 3 degrees of freedom

Correct Answer:- Option-D

Question59:-The mark of students of a class is normally distributed with mean 35 and standard deviation 5. The probability of a student mark between 25 and 45 is

A:-0.4772

B:-0.0228

C:-0.9544

D:-0.0456

Correct Answer:- Option-C

Question60:-The coefficient of variation of Poisson distribution with mean 16 is

A:-25%

B:-50%

C:-100%

D:-75%

Correct Answer:- Option-A

Question61:-Let X be $N_3(\mu, \Sigma)$ with $\Sigma = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Which of the following statements is/are true ?

(i) x_1 and x_2 are independent

(ii) x_1 and x_2 are not independent

(iii) (x_1, x_2) and x_3 are independent

(iv) (x_1, x_2) and x_3 are not independent

A:-Only (i and iii)

B:-Only (i and iv)

C:-Only (ii and iii)

D:-Only (ii and iv)

Correct Answer:- Option-C

Question62:-Sum of independent products of multivariate normal random vectors is

A:-Wishart Distribution

B:-Chi Square Distribution

C:-Hotelling T^2

D:-Mahalanobis D^2

Correct Answer:- Option-A

Question63:-Let X be $N_3(\mu, \Sigma)$ with $\mu' = [1 \ -2 \ 1]$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. Find the distribution of $X_1 - 2X_2 - X_3$

A:- $N_3(\mu, \Sigma)$

B:- $N(\mu, 6)$

C:- $N(4, 14)$

D:- $N(4, 6)$

Correct Answer:- Option-C

Question64:-Let x_1, x_2, \dots, x_{10} be a random sample of size 10 from $N_5(\mu, \Sigma)$. Then the distribution of $10(\bar{X} - \mu)' \Sigma^{-1}(\bar{X} - \mu)$ is

A:- χ_5^2

B:- χ_{10}^2

C:- $W(\Sigma, 10)$

D:- $W(\Sigma, 5)$

Correct Answer:- Option-A

Question65:-Let A_1 and A_2 are independently distributed with $W(\Sigma, 2)$ and $W(\Sigma, 5)$ respectively. Then the distribution of $A_1 + A_2$ is

A:- $W(\Sigma, 10)$

B:- $W(\Sigma, 2)$

C:- $W(\Sigma, 5)$

D:- $W(\Sigma, 7)$

Correct Answer:- Option-D

Question66:-Multivariate Behrin Fisher's problem is

A:-Testing of Mean vector is equal to a given vector when the covariance matrix is known

B:-Testing the equality of means of two multivariate populations when the covariance matrices are equal

C:-Testing the equality of means of two multivariate populations when the covariance matrices are not equal

D:-Testing of mean vector is equal to a given vector when the covariance

matrix is unknown

Correct Answer:- Option-C

Question67:-Consider the testing of equality of means of two multivariate normal populations $N(\mu^{(1)}, \Sigma)$ and $N(\mu^{(2)}, \Sigma)$. Then the relationship between Hotelling T^2 and Mahalanobis D^2 .

$$\text{A: } -T^2 = \frac{N_1 + N_2}{N_1 N_2} D^2$$

$$\text{B: } -T^2 = \frac{N_1 N_2}{N_1 + N_2} D^2$$

$$\text{C: } -T^2 = (N_1 + N_2 - 2) D^2$$

$$\text{D: } -D^2 = (N_1 + N_2 - 2) T^2$$

Correct Answer:- Option-B

Question68:-Suppose $X \sim N_p(\mu, \Sigma)$ and S is the sample covariance matrix of a sample of size N . then the Statistic $N(\bar{X} - \mu)' S^{-1} (\bar{X} - \mu)$ is

A:-Wishart distribution

B:-Normal distribution

C:-Hotelling T^2

D:-Chi square distribution

Correct Answer:- Option-C

Question69:-Suppose the random variables x_1 and x_2 have covariance matrix $\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$. Proportion of the total population variance explained by first principal component is

A:-1/7

B:-6/7

C:-5/7

D:-2/7

Correct Answer:- Option-B

Question70:-Consider an orthogonal factor model $X = M + LF + \epsilon$. Which of the following statements is/are true ?

i. Covariance matrix of specific factors is a diagonal matrix.

ii. Common factors and specific factors are uncorrelated.

iii. Variance of i^{th} variable is the i^{th} communality.

A:-Only (i and ii)

B:-Only (iii)

C:-all the above (i, ii and iii)

D:-Only (i)

Correct Answer:- Option-A

Question71:-The transition probability matrix of a Markov chain $\{x_n, n \geq 0\}$ having

three states 0, 1 and 2 is $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and the initial distribution is

$$P(X_0 = i) = \frac{1}{3}, i = 0, 1, 2.$$

The value of $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 0]$ is

A: -1/3

B: -1/54

C: -1/9

D: -2/3

Correct Answer:- Option-B

Question72:- Suppose a Markov chain $\{X_n\}$ have a state space $\{1, 2, 3, 4\}$ and

transition probability matrix is given by $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following statement

is/are true ?

(i) Markov chain $\{X_n\}$ is irreducible

(ii) Markov chain $\{X_n\}$ is aperiodic

(iii) Markov chain $\{X_n\}$ does not have a stationary distribution

A: -Only (i)

B: -Only (ii and iii)

C: -All of the above (i, ii and iii)

D: -Only (i and ii)

Correct Answer:- Option-D

Question73:- Let $\{X_n\}$ be a Markov chain having a state space $\{1, 2, 3, 4\}$ and

transition probability matrix is given by $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 1 \end{bmatrix}$. Find $\lim_{n \rightarrow \infty} P_{11}^n$

A: - ∞

B: -1

C: -0

D: -1/2

Correct Answer:- Option-C

Question74:- Let $\{X_n\}$ be a Markov chain having a state space $\{0, 1, 2\}$ and transition

probability matrix is given by $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$. Which of the following states are ergodic ?

A: - $\{0, 2\}$

B: -All the states

C: -1

D: -None of the states

Correct Answer:- Option-D

Question75:-Let $\{x_n\}$ be a Markov chain having a state space $\{1, 2, 3, 4, 5\}$ and

transition probability matrix is given by $P = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 1 \\ \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$

Which of the following statements is/are correct ?

- i. 1 and 4 are in the same communicating class.
- ii. 2 and 3 are in the same communicating class.
- iii. 2 and 5 are in the same communicating class.
- iv. 3 and 4 are in the same communicating class.

A:-Only (i and iii)

B:-Only (ii and iii)

C:-Only (iv)

D:-All the above (i, ii, iii and iv)

Correct Answer:- Option-A

Question76:-The interval between two successive occurrence of a Poisson process $N(t)$, $t \geq 0$ having parameter λ follows

A:-Exponential with mean λ

B:-Exponential with mean $\frac{1}{\lambda}$

C:-Poisson with parameter λ

D:-Geometric distribution with parameter λ

Correct Answer:- Option-B

Question77:-Suppose $\{x_n\}$ is a random walk on real line. A unit is moving from state i to $i + 1$ with probability $\frac{1}{2}$ and also from state i to $i-1$ with probability $\frac{1}{2}$. Then random walk is

- (i) Recurrent
- (ii) Null recurrent
- (iii) Aperiodic
- (iv) Ergodic

A:-All of the above (i, ii, iii and iv)

B:-Only (i and ii)

C:-Only (i, ii and iii)

D:-Only (iii)

Correct Answer:- Option-B

Question78:-Consider a birth and death process with λ as birth rate and μ as death rate. The probability of ultimate extinction when the death rate is greater than the birth rate is

A:- $\frac{\lambda}{\mu}$

B:-1

C:-< 1

D:-0

Correct Answer:- Option-B

Question79:-Consider a queuing model M/M/1 with interarrival time λ and service time μ . Which of the following statements is/are correct ?

- (i) Steady state distribution of Queue length exist if $\lambda < \mu$
- (ii) Steady state distribution of Queue length exist if $\lambda > \mu$
- (iii) Steady state distribution is given by Geometric distribution

A:-Only (i and iii)

B:-Only (ii and iii)

C:-Only (i)

D:-Only (iii)

Correct Answer:- Option-A

Question80:-Suppose $N(t)$ is a renewal process generated by exponential random variables X_1, X_2, \dots, X_n with parameter λ and $H(t)$ is a renewal function. Which of the following statements is/are true ?

- (i) $N(t)$ is a Poisson process
- (ii) Renewal function $H(t)$ is $\frac{t}{E(X)}$
- (iii) $\lim_{t \rightarrow \infty} \frac{H(t)}{t} = 0$ if $E(X) = \infty$

A:-Only (i)

B:-Only (ii and iii)

C:-Only (ii)

D:-All of the above (i, ii and iii)

Correct Answer:- Option-D

Question81:-Consider the simple linear regression model $y = \beta_0 + \beta_1 X + \epsilon$, Which of the following is a wrong statement ?

A:-Mean of Y is a linear function in X

B:-Variance of Y depends on the value of X

C:-Response variables are uncorrelated

D:-The sum of the residuals in simple linear regression models is always zero

Correct Answer:- Option-B

Question82:-Consider the simple linear regression model $y = \beta_0 + \beta_1 X + \epsilon$, Which of the following estimates are not unbiased ?

A:-Least square estimates of β_0 and β_1

B:-Maximum likelihood estimates of β_0 and β_1

C:-Least square estimates of σ^2

D:-Maximum likelihood estimates of σ^2

Correct Answer:- Option-D

Question83:-Consider the simple linear regression model $y = \beta_0 + \beta_1 X + \epsilon$ where ϵ 's are uncorrelated with mean zero and variance σ^2 , What is the value of $Cov(\widehat{\beta}_0, \widehat{\beta}_1)$?

A:- $\frac{-\bar{X}\sigma^2}{S_{XX}}$ where $S_{XX} = \sum (X_i - \bar{X})^2$

B:- $\frac{-\bar{X}\sigma^2}{S_{XY}}$ where $S_{XY} = \sum Y_i (X_i - \bar{X})$

C:- σ^2

D:- $\frac{\sigma^2}{S_{XX}}$ where $S_{XX} = \sum (X_i - \bar{X})^2$

Correct Answer:- Option-A

Question84:-Consider a multiple linear regression model $Y = X\beta + \epsilon$ with hat matrix $H = (h_{ij})$. The variance of i^{th} residual is

A:- σ^2

B:- $\sigma^2 h_{ii}$

C:- $\sigma^2 (1 - h_{ii})$

D:- $(1 - h_{ii})$

Correct Answer:- Option-C

Question85:-Consider a multiple linear regression model $Y = X\beta + \epsilon$, ϵ 's are uncorrelated with mean zero and variance σ^2 where the regression matrix X is

$\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 2 \end{bmatrix}$ and $\beta' = (\beta_0, \beta_1)$. Find variance of $\widehat{\beta}_1$

A:- $6\sigma^2$

B:- $\frac{\sigma^2}{6}$

C:- $5\sigma^2$

D:- $\frac{\sigma^2}{5}$

Correct Answer:- Option-D

Question86:-Which of the following is/are polynomial regression models ?

(i) $y = \beta_0 + \beta_1 X + \epsilon$

(ii) $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{12} X_1 X_2 + \epsilon$

(iii) $y = \beta_0 + \beta_1^2 X_1 + \beta_2^2 X_2 + \epsilon$

A:- Only (i)

B:- Only (i and ii)

C:- All of the above (i, ii and iii)

D:- None of the above

Correct Answer:- Option-B

Question87:-Suppose collinearity is present in a regression model then which of the statements is/are correct ?

- (i) Least Square estimate of Regression coefficients have large variances
- (ii) Eigenvalues of matrix $X'X$ is large
- (iii) Estimated coefficients have large variance inflation factor

A:-Only (i and iii)

B:-None of the above

C:-Only (i)

D:-Only (ii)

Correct Answer:- Option-A

Question88:-Suppose consider a logistic regression model with one regressor, the estimated increase in the odds ratio associated with a change of one unit in the predictor variable is

A:- β_1

B:- $e^{\hat{\beta}_1}$

C:- $\text{Log } \hat{\beta}_1$

D:- $\hat{\beta}_1$

Correct Answer:- Option-B

Question89:-Which of the following link function is suitable for Poisson Regression model ?

A:-Identity link function

B:-Log link function

C:-Reciprocal link function

D:-Logistic function

Correct Answer:- Option-B

Question90:-Consider a multiple linear regression model $Y = X\beta + \epsilon$, Where $V(\epsilon) = \sigma^2 V$ Then which of the following is correct ?

A:-Ordinary least square estimator is biased

B:-Ordinary least square estimator is minimum variance unbiased

C:-If V is a diagonal matrix but with unequal diagonal elements, then the observations y are correlated

D:-Generalised least square estimate is minimum variance unbiased

Correct Answer:- Option-D

Question91:-Let f be a Reimann integrable function on a closed interval $[a, b]$. Then which of the following statements is true ?

A:- f is continuous on $[a, b]$

B:- f is monotonic on $[a, b]$

C:- f^2 is Reimann integrable over $[a, b]$

D:-None of the above

Correct Answer:- Option-C

Question92:-Suppose f be defined on [0, 1] by

$$f(t) = 1 \quad 0 \leq t \leq 1/3$$

$$= 2 \quad 1/3 < t < 2/3$$

$$= 3 \quad 2/3 \leq t \leq 1$$

$\alpha = f$ and $\beta(t) = t^2$. Then which of the following is not true ?

A:-f is integrable with respect to α

B:-f is integrable with respect to β

C:- $f\beta$ is Reimann integrable

D:- $f\alpha$ is Reimann integrable

Correct Answer:- Option-A

Question93:-Evaluate the Integral $\int_0^1 \frac{x^2}{\sqrt{1-x^3}} dx$

A:- $\frac{\pi}{2}$

B:- $\frac{\pi}{4}$

C:- π

D:-0

Correct Answer:- Option-B

Question94:-The series $1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots$, is divergent when

A:- $P > 1$

B:- $0 < P < 1$

C:- $0 < P \leq 1$

D:- $P < 1$

Correct Answer:- Option-C

Question95:-Consider $f(x, y) = \frac{xy}{x^2+y^2}$ if $(x, y) \neq (0, 0)$
 $= 0$ if $(x, y) = (0, 0)$

Which of the following statement is true ?

A:- $f(x, y)$ is continuous at $(0, 0)$

B:-Partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at $(0, 0)$

C:-f is differentiable at $(0, 0)$

D:-None of the above

Correct Answer:- Option-B

$$\begin{bmatrix} 1 & 6 & 1 & 0 & 0 & 0 \\ 5 & 6 & 4 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 4 & 3 & 5 & 0 & 0 \\ 2 & 3 & 2 & 2 & 1 & 3 \\ 1 & 5 & 1 & 5 & 4 & 6 \end{bmatrix}$$

Question96:-Find the determinant of Matrix A =

A:-0

B:-57

C:-1710

D:-None of the above

Correct Answer:- Option-C

Question97:-Suppose A is an idempotent matrix of order n then which of the following statements are true ?

- (i) $A^r = A$ for being a positive integer
- (ii) A-I is idempotent
- (iii) I-A is Idempotent
- (iv) Rank (A) = trace (A)

A:-All the above (i, ii, iii and iv)

B:-Only (i, ii and iii)

C:-Only (iii and iv)

D:-Only (i, iii and iv)

Correct Answer:- Option-D

Question98:-Suppose $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ find the eigenvalues of A^4

A:-1, 3, -4

B:-1, 81, 256

C:-2, 1, -3

D:-1, 9, 16

Correct Answer:- Option-B

Question99:-Consider the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. What is the Geometric multiplicity and algebraic multiplicity of eigenvalue one ?

A:-1 and 2 respectively

B:-2 and 1 respectively

C:-1 and 1 respectively

D:-2 and 2 respectively

Correct Answer:- Option-A

Question100:-Suppose $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, which of the following statements is/are true ?

(i) A is positive definite matrix

(ii) A is positive semi definite matrix

(iii) A is non negative definite matrix

(iv) Quadratic form corresponding to matrix A is $X_1^2 + X_3^2$

A:-Only (iv) is true

B:-Only (ii, iii and iv)

C:-Only (ii and iv)

D:-Only (iii and iv)

Correct Answer:- Option-B