045/2025

PART – II

Total Number of Questions : 40	Maximum Marks : 200	Time : 3 Hours

INSTRUCTIONS (നിർദ്ദേശങ്ങൾ)

1. Question cum Answer Booklets are processed by electronic means. The following instructions are to be strictly followed to avoid invalidation of answer scripts.
   (ചോദ്യവും ഉത്തരവും അടങ്ങുന്ന ഈ ബുക്ക് ലെറ്റുകൾ ഇലക്ട്രോണിക് സാങ്കേതിക വിദ്യയുടെ സഹായത്തോടുകൂടെ മൂല്യനിർണ്ണയം നടത്തുന്നതിനാൽ ഇവ അസാധുവാകാതിരിക്കുവാൻ താഴെപ്പറയുന്ന നിർദ്ദേശങ്ങൾ പൂർണ്ണമായും പാലിക്കുക.)

2. The first page of this question cum Answer Booklet is an OMR data Sheet (Part I). All entries in the OMR sheet are to be made with blue or black ball point pen only.
   (ഈ പുസ്തകത്തിന്റെ ഒന്നാമത്തെ പേജ് ഒരു ഒ.എം.ആർ. ഡാറ്റാ ഷീറ്റാണ് (പാർട്ട് I). ഇത് നീലയോ, കറുപ്പോ നിറത്തിലെ ബോൾ പോയിന്റ് പേന ഉപയോഗിച്ച് മാത്രമേ പൂരിപ്പിക്കാവൂ.)

3. Make sure that register number is bubbled correctly and completely; no correction is permitted.
   (രജിസ്റ്റർ നമ്പർ രേഖപ്പെടുത്തുന്നതിനുള്ള കുമിളകൾ കൃത്യമായും പൂർണ്ണമായും കറുപ്പിച്ചിട്ടുണ്ടെന്ന് ഉറപ്പു വരുത്തുക. തിരുത്തലുകൾ അനുവദനീയമല്ല.)

4. Do not tamper the bar code printed on the OMR sheet and subsequent pages. Tampering of bar code will result in the invalidation of this booklet.
   (ഈ പുസ്തകത്തിൽ എവിടെയും പ്രിന്റ് ചെയ്തിരിക്കുന്ന ബാർ കോഡിൽ ഒരു കാരണവശാലും തിരുത്തലുകളോ, മാർക്കുകളോ പാടില്ല. ഇതിനു വിരുദ്ധമായി ചെയ്യുന്ന പക്ഷം ഈ പുസ്തകം അസാധുവാകുന്നതാണ്.)

5. Answers should be written with blue or black ball point pen only.
   (ഉത്തരങ്ങൾ നീലയോ, കറുപ്പോ നിറത്തിലെ ബോൾ പോയിന്റ് പേന ഉപയോഗിച്ച് മാത്രമേ എഴുതാവൂ.)

6. Do not write anything outside the margin of space provided for writing the answer and write only one line of answer between two lines.
   (പുസ്തകത്തിൽ ഉത്തരം എഴുതുവാൻ നൽകിയിരിക്കുന്ന സ്ഥലത്തിനു വെളിയിൽ യാതൊന്നും തന്നെ എഴുതുവാൻ പാടില്ല. രണ്ടു വരകൾക്കിടയിൽ ഒരു വരി ഉത്തരം മാത്രമേ എഴുതുവാൻ പാടുള്ളൂ.)

7. Rough work should be done only in the specific page provided with.
   (റഫ് വർക്കുകൾ ഇതിനായി നൽകിയിരിക്കുന്ന പേജിൽ മാത്രമേ ചെയ്യുവാൻ പാടുള്ളൂ.)

1. Determine whether the given relation $\mathcal{R}$ is an equivalence relation on the set $\mathbb{Z}\setminus\{0\}$. $n\,\mathcal{R}\,m$ if $nm > 0$, for all $n, m \in \mathbb{Z}\setminus\{0\}$. (5 Marks)

2. Prove the expression using basic Boolean operations on sets, $A'B + B'C' + AB + B'C = 1$. (5 Marks)

3. Compute the remainder of 8^{103} when divided by 13. [Hint: use Fermat's theorem] (5 Marks)

4. Find $\phi(18)$, where ϕ is the Euler's phi function. (5 Marks)

5. Form the quadratic equation whose roots are –2 and –3. (5 Marks)

6. Check whether the equation $x^2 - 5x + 5 = 0$ is a reciprocal equation or not?

(5 Marks)

7. Find all left cosets and right cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}$, the group of integers under usual addition. (5 Marks)

8. Consider the group $\mathbb{Z}_{24}$ under addition modulo 24. Is $\mathbb{Z}_{24}$ is a cyclic group? If so what are the generators of $\mathbb{Z}_{24}$? (5 Marks)

9. Find the value of the determinant using row reduction (5 Marks)

$$\begin{vmatrix} x & 2x & 3x \\ x+y & 3x+2y & 6x+3y \\ x+y+z & 4x+3y+2z & 10x+6y+3z \end{vmatrix}$$

10. Find the inverse matrix of $A = \begin{bmatrix} 1 & 1 & 2 \\ 9 & 2 & 0 \\ 5 & 0 & 3 \end{bmatrix}$ by using Cayley-Hamilton theorem. (5 Marks)

11. Show that $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + x_3 = 0,\ x_2 - x_4 = 0\}$ is a subspace of $\mathbb{R}^4$. Find the dimension of this subspace. (5 Marks)

12. Let V be the vector space of all 2 by 2 real matrices and let $T : V \to \mathbb{R}^2$ be the linear transformation given by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b+c,\ d)$, If $\mathfrak{B}$ is the standard basis for V and $\mathfrak{B}$ /=$\{(1,\ 1),\ (-1,\ 0)\}$ is a basis for $\mathbb{R}^2$, find the matrix of T relative to these bases. (5 Marks)

13. Show that every infinite set A contains an infinite sequence of distinct terms. Deduce that there is a function $f : A \to A$ that is one to one but not onto. (5 Marks)

14. Let (x_n) be a sequence of integers such that $x_n \neq x_{n+1}$ for all n. Prove or disprove the following statements.

    (a) (x_n) is a Cauchy sequence.

    (b) (x_n) cannot have a convergent subsequence. (5 Marks)

15. Determine whether the series $\sum_{n=0}^{\infty} \frac{2^n \sin^2(4n)}{3^n + \cos^2(2n)}$ converges or diverges.

(5 Marks)

16. Show that the series $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k^2}{2+k^3}$ is conditionally convergent.

(5 Marks)

17. Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$. (5 Marks)

18. For what values of a, m and b does the function

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

Satisfies the hypotheses of the mean value theorem on the interval [0, 2].

(5 Marks)

19. Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 9x^2 - 24x + 2$ on the interval $0 \le x \le 5$. (5 Marks)

20. Let $S_A = 15t^2 + 10t + 20$ and $S_B = 5t^2 + 40t$, $t \ge 0$, be the position functions of cars A and B that are moving along parallel straight lanes of a highway. At what instant of time do they have the same velocity?

Which car is ahead at this instant? (5 Marks)

21. Find the length of the curve $x = \left(\frac{y^3}{3}\right) + \frac{1}{4y}$ from $y = 1$ to $y = 3$. (5 Marks)

22. Using definite integrals find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \le x \le 4$ about x axis. (5 Marks)

23. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$. (5 Marks)

24. Consider a triangular plate with vertices (0, 0), (1, 0) and (1, 2) having constant density of 3 g/cm^2. Find the moment of the plate about y-axis.

(5 Marks)

25. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point (x_1, y_1, z_1) on it. (5 Marks)

26. Find a unit vector perpendicular to the vectors $3i + j + 2k$ and $2i - 2j + 4k$. Also find the angle between the given vectors. (5 Marks)

27. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. (5 Marks)

28. Find the equation of the plane passing through the point (2, –3, 1) and perpendicular to the line joining the point (3, 4, –1) and (3, –1, 5). (5 Marks)

29. Suppose that a geyser, centered at the origin of a polar co-ordinate system, sprays water in a circular pattern in such a way that the depth D of water that reaches a point at a distance of r feet from the origin in 1 hour is $D = \mathrm{k}re^{-r}$. Find the total volume of water that the geyser sprays inside a circle of radius R centered at the origin. (5 Marks)

30. Find the mass of the solid enclosed between the spheres $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 + z^2 = 8$, if the density is $\rho(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$. (5 Marks)

31. Evaluate $\int_{(1,2,2)}^{(2,3,4)} (yz - 1)\,dx + (z + xz + z^2)\,dy + (y + xy + 2yz)\,dz$. (5 Marks)

32. Determine $\int_C (y^3 - xy)\,dx + (xy + 3xy^2)\,dy$ where C is the boundary of the region in the first quadrant enclosed by $x = 0, y = 1 - x^2$ and $y = x^2$. (5 Marks)

33. Solve the differential equation : $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$. (5 Marks)

34. Solve the differential equation : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$. (5 Marks)

35. Find the general solution of the equation : $y'' - 5y' + 4y = 65\sin 2x$.

(5 Marks)

36. A manufacturing unit manufactures doors and windows. Keeping in view the market demand, the manufacturer intends to produce in all no more than 30 pieces per week. The market survey has shown that no more than 15 doors per week can be sold. The manufacturer is also committed to supply atleast 10 windows per week. The net profit on the sale of a window and door is Rs.100 per piece and Rs 150 per piece respectively. Formulate this situation as a Linear Programming Problem to maximize the total profit. (No need to solve, only formulation is needed.) (5 Marks)

37. Discuss the convergence of $\sum_{k=0}^{\infty} \frac{z^k}{(1-z^{k+1})(1-z^{k+2})}$, $|z| \neq 1$. (5 Marks)

38. Find all the poles and singularities of $f(z) = \frac{z-3-2i}{z^2-(4+3i)z+(1+5i)}$.

(5 Marks)

39. Verify whether $u(x, y) = x^2 - y^2$ is Harmonic. If so find its harmonic conjugate.

(5 Marks)

40. Evaluate $\int_{\Omega} x\, dz$ where Ω is the boundary of the square $[0, 1] \times [0, 1]$ with $\mathbb{C}$ considered as $\mathbb{R}^2$.

(5 Marks)