

086/2026

Question Booklet
Alpha Code

A

Question Booklet
Serial Number

Total No. of questions : 100

Time : 1 Hour 30 Minutes

Maximum : 100 Marks

INSTRUCTIONS TO CANDIDATES

1. The question paper will be given in the form of a Question Booklet. There will be four versions of question booklets with question booklet alpha code viz. A, B, C & D.
2. The Question Booklet Alpha Code will be printed on the top left margin of the facing sheet of the question booklet.
3. The Question Booklet Alpha Code allotted to you will be noted in your seating position in the Examination Hall.
4. If you get a question booklet where the alpha code does not match to the allotted alpha code in the seating position, please draw the attention of the Invigilator IMMEDIATELY.
5. The Question Booklet Serial Number is printed on the top right margin of the facing sheet. If your question booklet is un-numbered, please get it replaced by new question booklet with same alpha code.
6. The question booklet will be sealed at the middle of the right margin. Candidate should not open the question booklet, until the indication is given to start answering.
7. Immediately after the commencement of the examination, the candidate should check that the question booklet supplied to him contains all the 100 questions in serial order. The question booklet does not have unprinted or torn or missing pages and if so he/she should bring it to the notice of the Invigilator and get it replaced by a complete booklet with same alpha code. This is most important.
8. A blank sheet of paper is attached to the question booklet. This may be used for rough work.
9. **Please read carefully all the instructions on the reverse of the Answer Sheet before marking your answers.**
10. Each question is provided with four choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and darken the bubble corresponding to the question number using Blue or Black Ball Point Pen in the OMR Answer Sheet.
11. **Each correct answer carries 1 mark and for each wrong answer 1/3 mark will be deducted. No negative mark for unattended questions.**
12. No candidate will be allowed to leave the examination hall till the end of the session and without handing over his/her Answer Sheet to the Invigilator. Candidates should ensure that the Invigilator has verified all the entries in the Register Number Coding Sheet and that the Invigilator has affixed his/her signature in the space provided.
13. Strict compliance of instructions is essential. Any malpractice or attempt to commit any kind of malpractice in the Examination will result in the disqualification of the candidate.

086/2026

Maximum : 100 marks

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1. How many subsets does a finite set containing exactly 4 elements have?
(A) 4 (B) 16 (C) 8 (D) 32
2. Which combination of properties must a relation satisfy to be classified as a partial order relation?
(A) Reflexive, Symmetric, Transitive
(B) Irreflexive, Symmetric, Transitive
(C) Reflexive, Antisymmetric, Transitive
(D) Reflexive, Asymmetric, Transitive
3. A function $f: A \rightarrow B$ is considered a bijection if and only if it is :
(A) Only injective (one-to-one) (B) Only surjective (onto)
(C) Both injective and surjective (D) Neither injective nor surjective
4. What is the inverse function, $f^{-1}(x)$, for the function $f(x)=3x-5$?
(A) $\frac{x-5}{3}$ (B) $3x+5$ (C) $\frac{3}{x+5}$ (D) $\frac{x+5}{3}$
5. If the product of the roots of the quadratic equation $x^2-7x+k=0$ is 12, What is the value of k ?
(A) 12 (B) -12 (C) 7 (D) -7
6. In a mathematical induction proof, the step where we assume the statement holds true for an arbitrary natural number $n=k$ is known as the :
(A) Base case (B) Inductive hypothesis
(C) Inductive step (D) Conclusion
7. In how many distinct ways can the letters of the word "MATH" be arranged?
(A) 4 (B) 12 (C) 16 (D) 24
8. A committee of 3 people is to be formed from a group of 5 individuals. In how many ways can this committee be chosen?
(A) 10 (B) 15 (C) 20 (D) 60
9. In a right-angled triangle, the sine of an acute angle is defined as the ratio of :
(A) Adjacent side to Hypotenuse (B) Opposite side to Adjacent side
(C) Hypotenuse to Opposite side (D) Opposite side to Hypotenuse

10. The interior angles of a convex pentagon are in the ratio $2 : 3 : 3 : 5 : 5$. What is the measure of its largest interior angle?
- (A) 90° (B) 120° (C) 150° (D) 160°
11. A solid hemisphere has a curved surface area of $50\pi \text{ cm}^2$. What is its total surface area?
- (A) $50\pi \text{ cm}^2$ (B) $75\pi \text{ cm}^2$ (C) $100\pi \text{ cm}^2$ (D) $150\pi \text{ cm}^2$
12. Euler's formula for a convex polyhedron links the number of Faces (F), Vertices (V) and Edges (E) through which equation?
- (A) $F + E = V + 2$ (B) $V + E = F + 2$ (C) $F + V = E + 2$ (D) $F \times V = E \times 2$
13. If a right-angled triangle has sides of length 3 units, 4 units and 5 units, What is its area?
- (A) 6 sq units (B) 7.5 sq units (C) 10 sq units (D) 12 sq units
14. Let $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & \frac{4}{x} \end{bmatrix}$. If A is a scalar matrix, then the value of x is :
- (A) ± 2 (B) only -2 (C) only 2 (D) $x = 1$
15. Let A be a square matrix and A^* be its transposed conjugate. Then $A - A^*$ is :
- (A) Symmetric (B) Skew-symmetric
(C) Hermitian (D) Skew-Hermitian
16. If the row-reduced echelon form of an $n \times n$ matrix P is I_n , then P is :
- (A) Singular (B) Hermitian
(C) Non-singular (D) Symmetric
17. For what value of k does the system of equations $kx + 3y = 3$ and $5x - y = 2$ have unique solution :
- (A) $k = -15$
(B) $k = 15$
(C) All real values of k except -15
(D) No real value of k

18. P is a non singular matrix with an eigen value -5 . Then an eigenvalue of P^{-1} is :
- (A) $-\frac{1}{5}$ (B) $\frac{1}{5}$ (C) -5 (D) 5
19. The eigenvector corresponding to the eigenvalue 5 of the matrix $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ is :
- (A) $\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
20. The solution of $203x + 119y = 47$ is :
- (A) $x = 2, y = -1$ (B) No solution
(C) $x = -1, y = 2$ (D) $x = -3, y = 5$
21. If n is not a prime, then the prime divisor of n is :
- (A) $\leq \sqrt{n}$ (B) $\sqrt{n} + 1$ (C) $\leq \sqrt{n-1}$ (D) $\geq \sqrt{n}$
22. What is the least non negative residue of $5^{18} \pmod{7}$?
- (A) 5 (B) 7 (C) 1 (D) 8
23. If P is a prime number and a is an integer such that $\gcd(a, p) = 1$, then $a^{p-1} \pmod{p}$ is equal to :
- (A) 1 (B) 0 (C) $p - 1$ (D) a
24. What is the value of $25 + 37 \pmod{12}$?
- (A) $3 \pmod{12}$ (B) $6 \pmod{12}$ (C) $11 \pmod{12}$ (D) $2 \pmod{12}$
25. Which of the following is a pair of co-prime numbers ?
- (A) 8 and 12 (B) 2 and 20 (C) 4 and 9 (D) 14 and 42
26. If $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $C = f_{yy}(a, b)$, then $f(x, y)$ has a local maximum at (a, b) if :
- (A) $f_x(a, b) = 0, f_y(a, b) = 0, AC < B^2, A < 0$
(B) $f_x(a, b) = 0, f_y(a, b) = 0, AC = B^2, A > 0$
(C) $f_x(a, b) = 0, f_y(a, b) = 0, AC > B^2, A > 0$
(D) $f_x(a, b) = 0, f_y(a, b) = 0, AC > B^2, A < 0$

27. Find the value of $\int_0^1 x \ln x dx$:
- (A) $-\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$
28. The Maclaurin series of $f(x) = \frac{1}{1+x^2}$ is :
- (A) $\sum_{n=0}^{\infty} (-1)^n x^{2n}, |x| < 1$ (B) $\sum_{n=0}^{\infty} x^{2n}, |x| < 1$
(C) $\sum_{n=0}^{\infty} (-1)^n x^{2n+1}, |x| < 1$ (D) $\sum_{n=0}^{\infty} (-1)^n x^n, |x| < 1$
29. If $f(x) = f(2a-x)$, then $\int_0^{2a} f(x) dx$ is :
- (A) $\int_a^0 f(2a-x) dx$ (B) $2 \int_0^a f(x) dx$
(C) $-2 \int_0^a f(x) dx$ (D) 0
30. The function $u = \sqrt{x^2 + y^2}$ is homogeneous of degree :
- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 0
31. If $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = k$, then k is :
- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2
32. The asymptotes of $\frac{x^2}{9} - \frac{y^2}{16} = 1$ are :
- (A) $y = \pm \frac{4}{3}x$ (B) $y = \pm \frac{3}{4}x$ (C) $y = \pm \frac{9}{4}x$ (D) $y = \pm x$
33. The general solution of $\frac{dy}{dx} = y$ is :
- (A) $y = Cx$ (B) $y = Ce^x$ (C) $y = x + C$ (D) $y = e^{Cx}$

34. If $f(x) = x + \frac{1}{x}$, its minimum value for $x > 0$ is :
 (A) 1 (B) 2 (C) 3 (D) 4
35. The eccentricity of the conic $4x^2 + 9y^2 = 36$ is :
 (A) $\frac{\sqrt{5}}{3}$ (B) $\frac{2}{3}$ (C) $\frac{\sqrt{7}}{3}$ (D) $\frac{1}{3}$
36. If $u = x\phi\left(\frac{y}{x}\right)$, then $xu_x + yu_y$ is :
 (A) 0 (B) u (C) $2u$ (D) $3u$
37. If $F(x) = \int_x^3 t^2 dt$, then $F'(x)$ is :
 (A) x^2 (B) $-x^2$ (C) $3x^2$ (D) $-3x^2$
38. For which of the following functions are the conditions of Rolle's theorem not satisfied?
 (A) $f(x) = (x-a)^m (x-b)^n$, where m and n are positive integers
 (B) $f(x) = \log(\sin x)$, $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$
 (C) $f(x) = 1 - |x|$, $-1 \leq x \leq 1$
 (D) $f(x) = e^{-x} \sin x$, $0 \leq x \leq \pi$
39. If A and B are two nonempty subsets of $\mathbb{R}$ such that $a \leq b$ for all $a \in A$ and all $b \in B$. Which of the following statements are true ?
 (i) $\sup A \leq \sup B$
 (ii) $\sup A \leq \inf B$
 (A) Only (i) (B) Only (ii)
 (C) Both (i) and (ii) (D) None of the above
40. Which of the following sets are not countable?
 (i) The set $\mathbb{Q}$ of rational numbers
 (ii) The set $\mathbb{N}$ of natural numbers
 (iii) The set $\mathbb{N} \times \mathbb{N}$ of ordered pairs of natural numbers
 (A) Only (i) and (iii) (B) Only (iii)
 (C) Only (i) (D) None of the above

41. Let $\varepsilon, \delta > 0$ and $a \in \mathbb{R}$. Let $V_\gamma(a) = V_\varepsilon(a) \cap V_\delta(a)$, the intersection of the ε -neighbourhood and δ -neighbourhood of a . Choose the correct value of γ for defining the above γ -neighbourhood of a
- (A) $\gamma = |\varepsilon - \delta|$ (B) $\gamma = \min\{\varepsilon, \delta\}$
 (C) $\gamma = \max\{\varepsilon, \delta\}$ (D) Impossible to find γ
42. Among the sequences given below, identify the sequence which does not converge to the value 1?
- (A) $(n^{1/n})$ (B) $(2^{1/n})$ (C) $\left(\frac{1}{3n}\right)$ (D) $((2n)^{1/n})$
43. Which of the following statement is/are correct about sequences?
- (i) Every monotone sequence is convergent
 (ii) Every monotone sequence is bounded
 (iii) Every convergent sequence is bounded
 (iv) Every bounded monotone sequences is convergent
- (A) Only (i) and (iii) (B) Only (iii) and (iv)
 (C) Only (ii) and (iv) (D) Only (i) and (iv)
44. Choose the correct statement about the convergence of the following series :
- (i) $\sum \frac{1}{n^2}$
 (ii) $\sum \frac{1}{n!}$
 (iii) $\sum \frac{(-1)^n}{n}$
 (iv) $\sum \frac{1}{n}$
- (A) Only (iv) is divergent (B) Only (i) and (ii) are convergent
 (C) Only (i) is convergent (D) Only (i) and (iii) are convergent
45. Identify the divergent series from the following :
- (A) $\sum \left(\frac{4n+3}{5n+7}\right)^n$ (B) $\sum \left(\frac{1}{3}\right)^n$ (C) $\sum (2)^{-n}$ (D) $\sum \left(\frac{3n+4}{2n+7}\right)^n$
46. Which of the following functions is continuous at $x = 0$?
- (A) $\frac{1}{x}$ (B) $|x|$ (C) $\sin\left(\frac{1}{x}\right)$ (D) $\operatorname{sgn} x$
47. A function $f: [0,1] \rightarrow \mathbb{R}$ is defined. Find which of the following is not Riemann integrable if f is defined in $[0, 1]$ such that it is :
- (A) The Dirichlet's function (B) A continuous function
 (C) A Monotone function (D) A step function

48. Which of the following is not the polar representation of the complex number $\sqrt{2} - i\sqrt{2}$?
 (A) $(2, -\pi/4)$ (B) $(2, 7\pi/4)$ (C) $(2, 5\pi/4)$ (D) $(-2, 3\pi/4)$
49. Evaluate $\sqrt{3+4i}$:
 (A) $\pm(2+i)$ (B) $2+i$ (C) $2-i$ (D) $\pm(2-i)$
50. Which of the following can be an analytic function?
 (i) $f(x, y) = x^2 + y^2$
 (ii) $f(x, y) = x^2 + 2i$
 (iii) $f(x, y) = 4 + 2(x + y)$
 (A) Only (i) and (iii) (B) Only (ii)
 (C) Only (i) and (ii) (D) None of the above
51. If the vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $4\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar, then the value of p is :
 (A) 0 (B) 1 (C) 2 (D) 4
52. The unit vector normal to the surface $xyz^3 = 12$ at $(1, -1, 2)$ is :
 (A) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (B) $\frac{-\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{11}}$ (C) $\frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$ (D) $\frac{2\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{14}}$
53. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \sqrt{x^2 + y^2 + z^2}$. Then $\nabla\left(\frac{1}{r}\right)$ is :
 (A) $\frac{-\vec{r}}{r^3}$ (B) $\frac{\vec{r}}{r^3}$ (C) $\frac{-\vec{r}}{r^2}$ (D) $\frac{\vec{r}}{r^2}$
54. If $\vec{F} = (3xy - z)\hat{i} + (ax^2 + y)\hat{j} + (-x + 2z)\hat{k}$ is irrotational, then a is :
 (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2
55. The directional derivative of $\phi = (x^2 + y^2)^2 + (y^2 + z^2)^2 + (x^2 + z^2)^2$ at $(1, 1, 1)$ in the direction of the vector $2\hat{i} + \hat{j} + 2\hat{k}$ is :
 (A) $\frac{20}{3}$ (B) $\frac{40}{3}$ (C) $\frac{80}{3}$ (D) $\frac{100}{3}$

56. The value of the integral $\oint_C (4x^5 + 5y)dx + (2x + 3y^4)dy$, where C is the boundary of the region bounded by $y = 0$, $y = x$ and $x + y = 2$ is :
- (A) 1 (B) 2 (C) 3 (D) 6
57. The value of the integral $\iint_S [(y^2z^2 + 5yz^2)\hat{i} + (9x^2z + x^3z^4)\hat{j} + x^2y^3\hat{k}] \cdot d\vec{S}$, Where S is the closed surface bounding the region of the sphere $x^2 + y^2 + z^2 = 9$ lying in the first octant is :
- (A) 0 (B) 5 (C) 9 (D) 14
58. The order and degree of the differential equation $\left[\left(\frac{d^2y}{dx^2} \right)^3 - 1 \right]^2 - \frac{d^3y}{dx^3} = 0$ is respectively :
- (A) 2, 3 (B) 1, 3 (C) 2, 6 (D) 3, 1
59. The integrating factor of the differential equation, $x \frac{dy}{dx} = 3x + 4y$, $x \neq 0$ is :
- (A) x^{-4} (B) x (C) x^4 (D) $4x^{-1}$
60. The general solution of $y'' - 2y' + y = xe^x$ is :
- (A) $c_1e^{-x} + c_2e^x + xe^x$ (B) $c_1e^{-x} + c_2e^x + \frac{xe^x}{2}$
- (C) $c_1e^x + c_2xe^x + \frac{x^3e^x}{6}$ (D) $c_1e^x + c_2xe^x + \frac{x^3e^x}{3}$
61. The general solution of $y'' + 9y = 4 \sin 2x$ is :
- (A) $(c_1 + c_2x)e^{3x} + \frac{4}{9} \sin 2x$
- (B) $c_1e^{-3x} + c_2e^{3x} + 4 \sin 2x$
- (C) $c_1 \cos 3x + c_2 \sin 3x - \frac{4}{9} \sin 2x$
- (D) $c_1 \cos 3x + c_2 \sin 3x + \frac{4}{5} \sin 2x$

62. Which of the following statement(s) is/are true for the differential equation $(1 + e^x)dx + ydy = 0$?
- I. The given differential equation is exact.
- II. $x + e^x + \frac{y}{2} = c$ is the general solution of the given differential equation
- (A) I only (B) II only
(C) Both I and II (D) Neither I nor II
63. The differential equation $(xe^y + y)dx + (ye^x + x)dy = 0$ is :
- (A) Exact (B) Separable (C) Linear (D) None of these
64. Which of the following statement is *true* regarding the data measurement levels?
- (A) In interval level data absolute zero exists
(B) Ordinal level is the lowest data measurement level
(C) Nominal level data can be converted into a ratio level data
(D) Interval level data can be converted into ordinal level data
65. Which of the following statement is *always true*?
- (A) Absolute measures are independent of unit of measurements
(B) Quartile deviation is a measure of central tendency
(C) Unit of measurement of variance is the square of the unit of the original data set
(D) Kurtosis measures the symmetry of the data distribution
66. The mean and variance of the observations 10, 20, 30, 40 and 50 are 30 and 200 respectively. What are the mean and variance of the data set 100, 200, 300, 400 and 500 respectively?
- (A) 300 and 2000 (B) 300 and 20000
(C) 300 and 200 (D) 300 and 290
67. Marks obtained by the students in a class test were 40, 50, 60, 70 and 80. The computed mean and variance of the marks were 60 and 200 respectively. After revaluation, the new marks are 40, 50, 60, 60 and 90.
- The new mean and variance are respectively,
- (A) 60 and 280 (B) 60 and 200
(C) 60 and 220 (D) 60 and 300

68. Consider A, B, and C of three sets of observations, where, $A = \{-1, 0, 1\}$, $B = \{-2, -1, 0, 1, 2\}$ and $C = \{1, 2, 3, 4, 5\}$. Which of the following statements about the mean and variance of the observations in the sets *true*?
- (A) Variances of observations in A, B and C are the same
 (B) Means of observations in A and B are the same and Variances of observations in A and C are the same
 (C) Means of observations in A and B are the same and Variances of observations in B and C are the same
 (D) None of the above are true statements
69. Two distinguishable fair coins are tossed simultaneously. Given that the first one lands up head, the probability of the second one also lands up head is equal to :
- (A) $1/2$ (B) $1/4$ (C) $3/4$ (D) $2/3$
70. A fair die is thrown three times independently. The probability that 3 is the maximum value that appears among these throws is equal to :
- (A) $1/2$ (B) $1/216$ (C) $27/216$ (D) $19/216$
71. Consider two independent events A and B in the same sample space with $P(A) = 0.3$ and $P(B) = 0.4$. What is the probability that at least one of the events A, B occur?
- (A) 0.70 (B) 0.12 (C) 0.58 (D) 0.30
72. Let A, B and C be three events such that $P(A) = 3/5$, $P(B) = 1/2$ and $P(C) = 8/10$. Then, which of the following statement is *false*?
- (A) $P(A \cup B) \geq 3/5$ (B) $P(A \cap B \cap C) \leq 3/5$
 (C) $P(B \cap C) \leq 1/2$ (D) $P(A \cup B \cup C) \geq 8/10$
73. Two independent events A and B satisfy $P(A \cap B) = 1/10$, $P(A^c \cap B^c) = 1/3$ and $P(A) > P(B)$. Then $P(A)$ is :
- (A) $1/3$ (B) $3/5$ (C) $4/5$ (D) $2/7$
74. Three factories A, B and C supply respectively 70%, 20% and 10% of the bricks needed by a construction company. From the past experience it is known that 2%, 3% and 5% of the bricks supplied by these factories are defective. What is the probability that a brick found defective was supplied by C?
- (A) 0.20 (B) 0.30 (C) 0.25 (D) 0.15

75. A box contains two coins, one of which is fair and the other two-headed (both sides head). One coin is chosen at random and tossed twice. If two heads appear, then the probability that the chosen coin was two-headed is :
- (A) $1/3$ (B) $2/5$ (C) $2/3$ (D) $4/5$
76. The values of c such that $f(x) = \frac{c}{x}$ for $x = 1, 2, 3, \dots$ is a probability density function is :
- (A) 0 (B) $1/2$
(C) 2 (D) No such c exists
77. If $X \geq 0$ is a non-negative integer valued random variable such that $P(X = n) = p_n$, then which of the following statements about expected value of X is correct?
- (A) $E(X) = \sum_{n=0}^{\infty} P(X < n)$ (B) $E(X) = \sum_{n=1}^{\infty} P(X \geq n)$
(C) $E(X) = \sum_{n=0}^{\infty} P(X > n+1)$ (D) $E(X) = \sum_{n=1}^{\infty} P(X < n+1)$
78. Let X be a random variable with probability density function $f(x) = \begin{cases} 1+x & \text{if } -1 \leq x < 0, \\ 1-x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{elsewhere} \end{cases}$, The value of $F(1/2)$ is :
- (A) $3/4$ (B) $13/16$ (C) $7/8$ (D) $5/6$
79. If X and Y are independent random variables following Poisson distribution, then the conditional distribution of X given $X + Y$ is :
- (A) Poisson (B) Binomial (C) Uniform (D) Geometric
80. Let X be a random variable with the support $0, 1, 2, \dots$ with moment generating function $M_X(s) = e^{a(e^s - 1)}$. Then $P(X = 0)$ is :
- (A) a (B) $e^{a(e-1)}$ (C) e^{-a} (D) e^a
81. If X has a binomial distribution with parameters 10 and $1/2$, which is the most likely outcome?
- (A) $X = 0$ (B) $X = 5$ (C) $X = 8$ (D) $X = 10$

82. Which of the following statements are the necessary and sufficient conditions for the random variables X and Y to be independent?

(i) $f(x, y) = f(x) \cdot f(y)$

(ii) $E(XY) = E(X) \cdot E(Y)$

(iii) X and Y are uncorrelated

(iv) $E(X+Y) = E(X) \cdot E(Y)$

(A) Only statement (i) is correct

(B) Only statement (i) and (ii) are correct

(C) Only statements (i), (ii) and (iii) are correct

(D) All the statements are correct

83. Consider the joint density function :

$$f(x, y) = \begin{cases} x+y & \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are true?

(A) The marginal distribution of X is $f_x(x) = x + 0.5$, for $0 \leq x \leq 1$

(B) The marginal distribution of Y is $f_Y(y) = 2y$, for $0 \leq y \leq 1$

(C) X and Y are independent random variables

(D) None of the above statements are correct

84. If X and Y are random variables such that $Y = aX + b$, where a and b are constants, which of the following statements about the correlation ρ are true?

(i) $\rho_{X,Y} = 1$ if $a > 0$

(ii) $\rho_{X,Y} = -1$ if $a < 0$

(iii) $\rho_{X,Y}$ is dimensionless

(iv) $\rho_{X,Y} = 0$ if $-1 < a < 1$

(A) Only statement (i) is correct

(B) Only statement (ii) is correct

(C) Only statement (i), (ii) and (iii) are correct

(D) All the statements are correct

85. Let X be a random variable with $E(X^n) = \frac{1}{3}$ for $n = 1, 2, 3, \dots$. Define $Y = X^2$. Then the correlation between X and Y is :
- (A) $1/4$ (B) $1/2$ (C) $3/4$ (D) 1
86. Which of the following statements is wrong?
- (A) Correlation coefficient is the geometric mean of regression coefficients
- (B) Arithmetic mean of the regression coefficients is always greater than the correlation coefficient
- (C) If one of the regression coefficients is greater than 1, the other will be less than 1
- (D) Regression coefficients are independent of change of origin but not scale
87. The two lines of regressions Y on X and X on Y coincide when :
- (A) The correlation coefficient between X and Y is ± 1
- (B) The two regression coefficients b_{yx} and b_{xy} are same
- (C) The standard deviations σ_x and σ_y are such that $\sigma_y = 1/\sigma_x$
- (D) The correlation coefficient between X and Y is zero
88. If the two lines of regressions are $3X + 12Y - 19 = 0$ and $3Y + 9X - 46 = 0$, the mean of X and Y respectively are :
- (A) $19/3$ and $46/3$ (B) 4 and 3
- (C) $1/4$ and $1/3$ (D) 5 and $1/3$
89. Which of the following is/are true for simple random sampling?
- (i) All units need to be homogeneous
- (ii) If the population is heterogeneous then population should be stratified and select sample from each strata
- (iii) Variance of sample mean is always less than all other sampling schemes
- (iv) Only first unit is selected at random
- (A) (i) only (B) (i) and (ii) only
- (C) (iii) and (iv) only (D) (i), (ii) and (iv) only

90. Which of the following is/are correct in sampling theory?
- (i) Systematic sampling is applied when population is heterogeneous only
 - (ii) In simple random sampling, every member of the population has an equal and independent chance of being selected
 - (iii) In stratified random sampling, the population is first divided into mutually exclusive, homogeneous subgroups (strata) based on shared characteristics and take samples from each strata.
 - (iv) Stratified random sampling is more efficient than cluster sampling when units are homogeneous
- (A) (i) only (B) (ii) and (iii) only
(C) (iii) and (iv) only (D) (iv) only
91. Which of the following tests is/are based only on stratum variance and stratum size in the population?
- (i) Equal allocation
 - (ii) Proportional allocation
 - (iii) Neyman allocation
 - (iv) Optimum allocation
- (A) (i) and (ii) only (B) (ii) and (iv) only
(C) (iii) and (iv) only (D) (iii) only
92. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from Normal distribution $N(\mu, \sigma^2)$. If $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$, then what is the distribution of $\sqrt{n}(\bar{X} - \mu)/S$?
- (i) chi-square (n)
 - (ii) $t(n)$
 - (iii) $t(n-1)$
 - (iv) Chi-square ($n-1$)
- (A) (i) only (B) (ii) only
(C) (iii) only (D) (i) and (iv) only
93. Which of the following conditions is/are correct for variance under different allocations in random sampling?
- (i) $V_{optimum} \leq V_{proportional} \leq V_{simple random}$
 - (ii) $V_{simple random} \leq V_{proportional} \leq V_{optimum}$
 - (iii) $V_{proportional} \leq V_{optimum} \leq V_{simple random}$
 - (iv) $V_{optimum} \leq V_{simple random} \leq V_{proportional}$
- (A) (i) only (B) (ii) only (C) (iii) only (D) (iv) only

94. Which of the following is/are the properties of Uniformly minimum variance unbiased estimator?
- (i) The estimator is sufficient
 - (ii) The estimator has least variance
 - (iii) The estimator is complete
 - (iv) The estimator is unbiased
- (A) (i) and (ii) only (B) (ii) and (iii) only
(C) (iii) only (D) All of the above
95. Which of the following is/are true for sampling distributions?
- (i) Square of standard Normal distribution is chi- square distribution
 - (ii) $F(1, n)$ and $[t(n)]^2$ have the same distribution
 - (iii) If F has $F(m, n)$ distribution, then $1/F$ has $F(1/m, 1/n)$ distribution
 - (iv) Square of standard normal distribution is $F(1, 1)$
- (A) (i) only (B) (i) and (ii) only
(C) (iii) only (D) (iv) only
96. Which is/are true for testing of hypothesis?
- (i) If a complete statistic T exist for a family of distributions, the Neyman–Pearson most powerful test is a function of T .
 - (ii) If a sufficient statistic T exist for a family of distributions, the Neyman-Pearson most powerful test is a function of T .
 - (iii) Neyman Pearson most powerful test is always a function of minimal sufficient statistic
 - (iv) Neyman Pearson most powerful test is always a functions of complete statistic
- (A) (i) only (B) (i) and (ii) only
(C) (ii) only (D) (iv) only
97. Which of the following properties is/are always true for a maximum likelihood estimator (MLE)?
- (i) MLE is always unbiased
 - (ii) MLE is always consistent
 - (iii) MLE is always UMVUE
 - (iv) MLE is always sufficient
- (A) (i) and (ii) only (B) (ii) and (iii) only
(C) (iii) and (iv) only (D) (ii) and (iv) only

98. Which of the following statistic is used to test hypothesis using likelihood ratio test (L_0 is the maximized likelihood of the null model and L_1 is the maximized likelihood of the alternative model)?

(i) $-\frac{1}{2}\log(L_0 / L_1)$

(ii) $-2\log(L_0 / L_1)$

(iii) $-2\log(L_1 / L_0)$

(iv) $-\frac{1}{2}\log(L_1 / L_0)$

(A) (i) only (B) (ii) only (C) (iii) only (D) (iv) only

99. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from Bernoulli distribution $Bernoulli(p)$. Then what is the distribution of $\frac{\sum_{i=1}^n X_i - np}{\sqrt{npq}}$, as $n \rightarrow \infty, 0 < p < 1, q = 1 - p$:

(A) Binomial (n, p)

(B) Bernoulli (l, p)

(C) Standard Normal (0, 1)

(D) Normal (np, npq)

100. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from Normal distribution

$N(\mu, \sigma^2)$. If $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$ and $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$. Then which of the statements is/are correct?

(i) $\bar{X}$ and S^2 are independent

(ii) $\sum_{i=1}^n X_i$ and $\sum_{i=1}^n (X_i - \bar{X})^2$ are independent

(iii) $\bar{X}$ and $\frac{nS^2}{\sigma^2}$ are independent

(iv) $\frac{\bar{X}}{S^2}$ and $\frac{nS^2}{\sigma^2}$ are independent

(A) (i) and (ii) only

(B) (i), (ii) and (iii) only

(C) (iii) and (iv) only

(D) (iv) only

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